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Global existence and asymptotic behavior for a time fractional reaction–diffusion system

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ABSTRACT

This paper is concerned with the existence of global in time solutions of a time fractional reaction–diffusion system with time fractional derivatives. Furthermore, the large time behavior of bounded solutions is investigated. Our method of proof relies on a maximal regularity result for fractional linear reaction–diffusion equations that has been derived by Bajlekova (2001).

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1. Introduction

This paper is concerned with the existence of global in time positive solutions of the time fractional reaction–diffusion system with a balance law

$$\begin{cases} {}^C D_t^\beta u - d\Delta u = -uf(v), & \text{in } \Omega \times \mathbb{R}^+, \\ {}^C D_t^\beta v - \Delta v = uf(v), & \text{in } \Omega \times \mathbb{R}^+, \end{cases} \tag{1}$$

supplemented with the boundary and initial conditions

$$\frac{\partial u}{\partial \eta}(x,t) = \frac{\partial v}{\partial \eta}(x,t) = 0 \quad \text{on } \partial\Omega \times \mathbb{R}^+, \tag{2}$$

$$u(x,0) = u_0(x), \qquad v(x,0) = v_0(x) \quad \text{in } \Omega, \tag{3}$$

where Ω is a regular bounded domain in $\mathbb{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial\eta}$ denotes the normal derivative on $\partial\Omega$, Δ stands for the Laplacian operator, d is the diffusion constant, u_0 and v_0 are nonnegative functions, ${}^C D_t^\beta$, for $\beta \in (0,1)$, is the Caputo fractional derivative of order β.

Concerning the nonlinearity f, we assume that there exist positive constants M_1 and M_2 and a real number $p \geq 1$ such that

$$0 \leq f(v) \leq M_1 |v|^p + M_2, \tag{4}$$

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and for all $|v|,\ |\tilde{v}| \le R$, there exists a positive number L such that

$$|f(v) - f(\tilde{v})| \le L|v - \tilde{v}|. \tag{5}$$

Before presenting our results with their proofs, let us dwell upon some literature concerning reaction–diffusion systems with a balance law. When considering the time evolution of the spatio-temporal concentrations of the species U and V of the molecular combination

$$mU + nV \to (n+1)V,$$

one is lead to the reaction–diffusion system

$$\begin{cases} u_t - a\Delta u = -u^m v^n, & x \in \Omega,\ t > 0, \\ v_t - b\Delta v = u^m v^n, & x \in \Omega,\ t > 0, \end{cases} \tag{6}$$

where Ω is the medium in which the molecular combination takes place. System (6) has been successfully studied by Masuda [1], Martin, Hollis and Pierre [2], Kanel and Kirane [3] and Abdelmalek and Kouachi for a very general situation [4]; Ahmad et al. [5] studied the space fractional system

$$u_t + (-\Delta)^{\frac{\alpha}{2}} u = -f(u, v), \quad x \in \mathbb{R}^n,\ t > 0,$$
$$v_t - (-\Delta)^{\frac{\beta}{2}} v = f(u, v), \quad x \in \mathbb{R}^n,\ t > 0,$$

via the duality argument.

Here, we want to see the influence of the fractional time derivatives on the behavior of the solutions.

Fractional derivatives occur when the system takes place in an irregular or fractal medium. Needless to say that fractional reaction–diffusion equations and system are currently extensively studied not only for the mathematical side but also for their potential applications [6–8].

In this paper, we first prove that system (1)–(2)–(3) admits global solutions by relying on a maximal regularity result derived by Bajlekova. We also derive the large time behavior of bounded solutions.

2. Preliminary results

In this section, we introduce some basic definitions of fractional calculus which are used here after, see [9].

Definition 2.1. For an integrable function f, the Riemann–Liouville integral of order $\beta \in (0, 1)$ is defined by

$$J_t^{\beta} f(t) := \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} f(s)\, ds, \quad t > 0,$$

where Γ is the Euler Gamma function.

Definition 2.2. For an absolutely continuous function f, the Caputo fractional derivative of order $\beta \in (0, 1)$ is

$${}^{C}D_t^{\beta} f(t) := D_t^{\beta}(f(t) - f(0)), \quad t > 0, \tag{7}$$

where D_t^{β} is the Riemann–Liouville fractional derivative of order β given by

$$D_t^{\beta} f(t) := \frac{d}{dt} J_t^{1-\beta} f(t). \tag{8}$$

In particular, if $f(0) = 0$ we have

$${}^{C}D_t^{\beta} f(t) = D_t^{\beta} f(t), \quad t > 0. \tag{9}$$

Lemma 2.3. *It holds*

$$J_t^{\beta}\, {}^{C}D_t^{\beta} f(t) = f(t) - f(0), \quad t > 0, \tag{10}$$

and

$${}^{C}D_t^{\beta} J_t^{\beta} f(t) = f(t), \quad t > 0. \tag{11}$$

Definition 2.4. We denote by A the realization of $-\Delta$ with homogeneous Neumann boundary conditions in $L^2(\Omega)$.

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