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Parameters estimation for a new anomalous thermal diffusion model in layered media

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ABSTRACT

In this paper, we study an inverse problem of parameters estimation for a new time-fractional heat conduction model in multilayered medium. In the anomalous thermal diffusion model, we consider the fractional derivative boundary conditions and the conduction obeys modified Fourier law with Riemann–Liouville fractional operator of different order in each layer. For the direct problem, we construct an effective finite difference scheme by using the balance method to deal with the discontinuity interface. For the inverse problem, we apply the nonlinear conjugate gradient (NCG) method with different conjugated coefficients to simultaneously identify the fractional exponent in each layer. Finally, we use experimental data to verify the effectiveness of the proposed technique, in which the Jacobian matrix is achieved by a derivative-free approach. We analyze the sensitivity coefficients and the convergence behaviors of the NCG algorithm. The simulation results confirm that the fractional heat conduction model with estimated parameters gives a more accurate fitting than the classical counterpart and the NCG method is a feasible and effective technique for the inverse problem of parameters estimation in fractional model.

1. Introduction

Anomalous sub-diffusion model governed by using time fractional operator has been a rapidly growing field of research. In general, the anomalous diffusion model with time fractional derivative provides a more adequate and accurate description in capturing the memory and hereditary properties inherited by the transport process in heterogeneous porous media [1,2]. From the point of stochastic view, fractional in time model sub-diffusion, related to long power-law waiting times between particle jumps. Specifically, in the anomalous sub-diffusive systems, the mean square displacement of the particles is no longer linear in time. Instead, sub-diffusion motion is characterized by an asymptotic long time behavior of the mean square displacement of the form [3]

$$\langle x^2(t)\rangle \sim t^{\alpha}, \quad t \longrightarrow \infty, \tag{1}$$

with $0 < \alpha < 1$ being the anomalous diffusion exponent.

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To capture the feature of the anomalous heat conduction process in media with layered structure, we employ the modified constitutive relation [4–7]

$$\mathbf{q}(x,t) = -\frac{1}{\Gamma(\alpha)}\frac{\partial}{\partial t}\int_0^t (t-s)^{\alpha-1}\left[k(x)\frac{\partial u(x,s)}{\partial x}\right]ds, \quad 0<\alpha<1, \tag{2}$$

in which, the time-non-local dependence between the thermal flux density and the temperature gradient with the 'long-tail' power kernel is established. In this work, the fractional exponent α is focus of our concern since it is of great significance in characterizing the anomalous thermal diffusion process. It should be noted that researches on the fractional diffusion in composite media are still limited [8–12]. In the new fractional thermal diffusion model, we assume that the heat conduction process obeys different modified Fourier law in each layer of the composite media. That is, the fractional exponents α in the flux term (2) are different for each layer. The main objective of this paper aims at presenting an effective and efficient algorithm for identifying the fractional exponent α in composite material.

In fact, the measurement of some quantities of interest in a fractional model presents a very difficult and challenging inverse problem. However, the studies on the inverse problem for fractional partial differential equation models with experimental data have been rarely reported. Jiang et al. [11] proposed a Caputo time-fractional heat conduction model for a three layered medium and the fractional order was numerically identified. Jiang et al. [12] proposed a numerical approach for estimating the relaxation parameters and fractional derivative for a thermal wave model in bi-layered spherical tissue. A Bayesian method was applied to identify parameters in the Zener model of viscoelasticity based on a generalized fractional element network in [13]. Liu et al. [14] combined a modified hybrid simplex search and a particle swarm optimization to estimate the parameter in a fractional dynamical model arising from biological systems. Chen et al. [15] presented a coupled method with Tikhonov regularization to identify the source term in a spatial fractional anomalous diffusion model. Ghazizadeh [16] performed an inverse analysis for simultaneous estimation of the relaxation time and the order of the fractional derivative in a fractional single-phase-lag heat equation. Wang and Liu [17] considered an image deblurring problem governed by the time-fractional diffusion process, in which an iteration method with the total variation regularization technique was used to achieve the minimizer of the corresponding cost functional. For the theoretical consideration and efficient numerical implementation on the regularized techniques for inverse problem in fractional partial differential equation models, we refer to [18–27].

The main structure of this paper is organized as follows: We firstly present a new anomalous thermal diffusion model with different fractional exponents in the layered medium and the mathematical formulation of the inverse problem for identifying the fractional derivative indices in Section 2. In Section 3, for the direct problem, we derive an implicit finite difference method by applying the balance method to deal with the interfacial point. While, for the inverse problem, we apply the nonlinear conjugate gradient method with different conjugate coefficients to give an effective iterative procedure. In Section 4, we utilize the experimental data from a carbon–carbon composite structure to get the numerical inversion of the fractional derivative exponents. The corresponding relative sensitivity coefficients and simulation results are analyzed and drawn graphically. Finally, we present conclusions and discussions in Section 5.

2. Mathematical model

For the simplicity of presentation but without loss of generality, we consider the composite material with a three layered structure. The governing equation for anomalous heat conduction can be described as

$$\epsilon(x)\frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left({}_0^{RL}\mathcal{D}_t^{1-\alpha_p}\left[k(x)\frac{\partial u(x,t)}{\partial x}\right]\right), \quad 0<\alpha_p<1, \quad L_{p-1}<x<L_p, \quad 0<t<T, \tag{3}$$

with

$$\epsilon(x) = \rho_p c_p, \quad \text{for } L_{p-1}<x<L_p, \tag{4}$$

$$k(x) = k_p, \quad \text{for } L_{p-1}<x<L_p, \tag{5}$$

where $p = 1, 2, 3$, $u(x,t)$ is the temperature distribution function, ρ_p, c_p, k_p are the corresponding specific heat capacity, density, and thermal conductivity in each layer; ${}_0^{RL}\mathcal{D}_t^{1-\alpha}$ represents the Riemann–Liouville fractional derivative of order $1-\alpha$ with respect to t defined by [1–3]

$${}_0^{RL}\mathcal{D}_t^{1-\alpha}u(x,t) = \frac{1}{\Gamma(\alpha)}\frac{\partial}{\partial t}\int_0^t (t-s)^{\alpha-1}u(x,s)ds, \quad 0<\alpha<1.$$

To close the system (3)–(5), we present the following boundary, interfacial and initial conditions

- left boundary condition of the 1st layer

$$-{}_0^{RL}\mathcal{D}_t^{1-\alpha_1}\left[k(x)\frac{\partial u(x,t)}{\partial x}\right]_{x=L_0} = q_0, \tag{6}$$

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