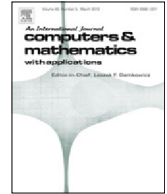




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A divide-and-conquer fast finite difference method for space–time fractional partial differential equation

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ABSTRACT

Fractional partial differential equations (FPDEs) provide better modeling capabilities for challenging phenomena with long-range time memory and spatial interaction than integer-order PDEs do. A conventional numerical discretization of space–time FPDEs requires $\mathcal{O}(N^2 + MN)$ memory and $\mathcal{O}(MN^3 + M^2N)$ computational work, where N is the number of spatial freedoms per time step and M is the number of time steps.

We develop a fast finite difference method (FDM) for space–time FPDE: (i) We utilize the Toeplitz-like structure of the coefficient matrix to develop a matrix-free preconditioned fast Krylov subspace iterative solver to invert the coefficient matrix at each time step. (ii) We utilize a divide-and-conquer strategy, a recursive direct solver, to handle the temporal coupling of the numerical scheme. The fast method has an optimal memory requirement of $\mathcal{O}(MN)$ and an approximately linear computational complexity of $\mathcal{O}(NM(\log N + \log^2 M))$, without resorting to any lossy compression. Numerical experiments show the utility of the method.

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1. Introduction

FPDEs provide better modeling capabilities for challenging phenomena with long-range time memory and spatial interactions than integer-order PDEs do [1–6]. However, due to the nonlocal nature of fractional differential operators, the numerical schemes of FPDEs give rise to dense stiffness matrices and/or long tails in time or a combination of both. Traditionally, a time-marching solution method with direct solvers has been used to solve these schemes that require $\mathcal{O}(N^2 + MN)$ memory and $\mathcal{O}(MN^3 + M^2N)$ overall computational work, where N is the number of spatial unknowns at each time step and M is the number of time steps. This is deemed computationally very expensive in terms of computational complexity and memory requirement, especially for problems in multiple space dimensions [7,8]. The significantly increased computational complexity and memory requirements is one of the main reasons why FPDE models have not been used widely.

In this paper we develop a divide-and-conquer fast FDM for space–time FPDE: (i) We utilize the Toeplitz-like structure of the coefficient matrix to develop a matrix-free preconditioned fast Krylov subspace iterative method to invert the coefficient matrix at each time step. (ii) We utilize a divide-and-conquer strategy, a recursive direct solver, to handle the temporal coupling of the numerical scheme. The fast method has an optimal memory requirement of $\mathcal{O}(MN)$ and an approximately linear computational complexity of $\mathcal{O}(NM(\log N + \log^2 M))$, without resorting to any lossy compression. Numerical experiments show the utility of the method.

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