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Fitted reproducing kernel Hilbert space method for the solutions of some certain classes of time-fractional partial differential equations subject to initial and Neumann boundary conditions

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ABSTRACT

Latterly, many problems arising in different fields of science and engineering can be reduced, by applying some appropriate discretization, to a series of time-fractional partial differential equations. Unlike the normal case derivative, the differential order in such equations is with a fractional order, which will lead to new challenges for numerical simulation. The purpose of this analysis is to introduce the reproducing kernel Hilbert space method for treating classes of time-fractional partial differential equations subject to Neumann boundary conditions with parameters derivative arising in fluid-mechanics, chemical reactions, elasticity, anomalous diffusion, and population growth models. The method provides appropriate representation of the solutions in convergent series formula with accurately computable components. Numerical experiments with different order derivatives degree are performed to support the theoretical analyses which are acquired by interrupting the n-term of the exact solutions. Finally, the obtained outcomes showed that the proposed method is competitive in terms of the quality of the solutions found and is very valid for solving such time-fractional Neumann problems.

1. Introduction

The one-dimensional nonlinear fractional models, which are representative in the time-fractional partial differential equations (PDEs), have been studied to describe numerous realism matters successfully not only in physics, but also in engineering, biology, economics, and other sciences [1–5]. Such models are utilized extensively by many experts to explain their complicated structures easily, simplified the controlling design without any loss of hereditary behaviors as well as create nature issues closely understandable for these phenomena. Consequently, fractional derivatives provide more accurate models of realism problems than integer-order derivatives; they are actually found to be a suitable tool to describe certain physical and engineering problems including reaction diffusion models, dynamical mathematical models, electrical circuits models, signal processing models, and so on [6–13]. Developing analytical and numerical methods for the solutions of time-fractional PDEs is a very important task. Indeed, it is difficult to obtain exact solutions form in general for most cases. Therefore, attempts have been made to propose analytical methods that approximate the exact solutions of such equations [6–35].

The purpose of this analysis is to investigate and implement a computational iterative method, the reproducing kernel Hilbert space method (RKHSM), in finding approximate solutions for various certain classes of Neumann time-fractional

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PDEs with parameters derivative in the sense of Riemann–Liouville and Caputo fractional derivatives. More specifically, we consider two types of time-fractional models in the fractional operator form.

(I) The following general form of nonlinear second-order time-fractional PDE with constant coefficients:

$$\partial_t u(x,t) = \left(A_\alpha\, {}_0\mathcal{D}_t^{1-\alpha} + B_\beta\, {}_0\mathcal{D}_t^{1-\beta}\right)\left[\partial_{x^2}^2 u(x,t)\right] - \left(C_\alpha\, {}_0\mathcal{D}_t^{1-\alpha} + D_\beta\, {}_0\mathcal{D}_t^{1-\beta}\right)[u(x,t)] + f(x,t,u(x,t)), \quad (1)$$

subject to the following initial and Neumann boundary conditions:

$$\begin{aligned} &u(x,0) = \omega(x), \\ &\partial_x u(0,t) = \upsilon_1(t), \qquad \partial_x u(L,t) = \upsilon_2(t), \end{aligned} \quad (2)$$

where $0 < \alpha, \beta < 1, 0 \le t \le T \in \mathbb{R}, 0 \le x \le L \in \mathbb{R}, A_\alpha, B_\beta, C_\alpha, D_\beta$ are nonnegative real constants, f is continuous real-valued function, u is an unknown function to be determined. Here, ${}_0\mathcal{D}_t^{1-\gamma}$ denote the Riemann–Liouville time-fractional derivatives operator of order $1-\gamma$ of a function $u(x,t)$ and defined as

$${}_0\mathcal{D}_t^{1-\gamma} u(x,t) = \frac{1}{\Gamma(\gamma)}\partial_t \int_0^t (t-\tau)^{\gamma-1} u(x,\tau)\, d\tau, \quad 0 < \tau < t,\ 0 < \gamma < 1. \quad (3)$$

To specify more, the time-fractional PDE of Eq. (1) consists of the following well-known certain equations as special cases:

- If $B_\beta = C_\alpha = D_\beta = 0$ and $f(x,t,u(x,t)) = g(x,t)$, then we obtain the time-fractional heat equation. The heat equation is derived from Fourier's law and conservation of energy, it is used in describing the distribution of heat or variation in temperature in a given region over time [14,15].
- If $B_\beta = C_\alpha = 0$, then we obtain the time-fractional cable equation. The cable equation is derived from the cable equation for electrodiffusion in smooth homogeneous cylinders, it is occurred due to anomalous diffusion and is used in modeling the ion electrodiffusion at the neurons [16,17].
- If $C_\alpha = D_\beta = 0$ and $f(x,t,u(x,t)) = g(x,t)$, then we obtain the time-fractional modified anomalous subdiffusion equation. The modified anomalous subdiffusion equation is derived from the neural cell adhesion molecules, it is used for describing processes that become less anomalous as time progresses by the inclusion of a second fractional time derivative acting on the diffusion term [18,19].

(II) The following general form of nonlinear second-order time-fractional PDE with variable coefficients:

$$\partial_{t^\alpha}^\alpha u(x,t) = \left(\partial_{x^2}^2 p(x,u(x,t)) - \partial_x q(x,u(x,t))\right)[u(x,t)] + f(x,t,u(x,t)), \quad (4)$$

subject to the following initial and Neumann boundary conditions:

$$\begin{aligned} &u(x,0) = \omega_1(x), \\ &\partial_x u(0,t) = \upsilon_1(t), \qquad \partial_x u(L,t) = \upsilon_2(t), \end{aligned} \quad (5)$$

where $0 < \alpha < 1, 0 \le t \le T \in \mathbb{R}, 0 \le x \le L \in \mathbb{R}, f$ is continuous real-valued functions, u is an unknown function to be determined. Here, $\partial_{t^\alpha}^\alpha = \partial^\alpha/\partial t^\alpha$ denote the Caputo time-fractional derivatives operator of order α of a function $u(x,t)$ and defined as

$$\partial_{t^\alpha}^\alpha u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha}\, \partial_\tau u(x,\tau)\, d\tau, \quad 0 < \tau < t,\ 0 < \alpha < 1. \quad (6)$$

To specify more, the time-fractional PDE of Eq. (4) consists of the following well-known certain equations as special cases:

- If $p(x,u(x,t)) = 1$, $q(x,u(x,t)) = 0$, and $f(x,t,u(x,t)) = -E_\alpha u(x,t) + g(x,t)$, then we obtain the time-fractional reaction subdiffusion equation. The reaction subdiffusion equation appears in many different areas of chemical reactions, such as exciton quenching, recombination of charge carriers or radiation defects in solids, and predator–pray relationships in ecology [20,21].
- If $f(x,t,u(x,t)) = 0$, then we obtain the time-fractional Fokker–Planck equation [22,23]. The Fokker–Planck equation arises in many phenomena in plasma and polymer physics, population dynamics, neurosciences, nonlinear hydrodynamics, pattern formation, and psychology [24–27].
- If $p(x,u(x,t)) = 1$, $q(x,u(x,t)) = 0$, and $f(x,t,u(x,t)) = F_\alpha u(x,t)(1 - G_\alpha u^n(x,t)) + g(x,t)$, then we obtain the time-fractional Fisher's equation when $n = 1$ and the time-fractional Newell–Whitehead equation when $n = 2$. The Fisher's equation is used to describe the population growth models [28,29], whilst, the Newell–Whitehead equation arises in fluid dynamics model and capillary–gravity waves [30,31].

The RKHSM is a numerical, as well as, analytical technique for solving a large variety of ordinary and PDEs associated to different kinds of order derivatives degree, and usually provides the solutions in terms of rapidly convergent series with components that can be elegantly computed. The advantages of the utilized approach lie in the following; firstly, it

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