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Creep constitutive models for viscoelastic materials based on fractional derivatives

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A B S T R A C T

To describe the time-dependent creep behavior of viscoelastic material, fractional constitutive relation models which are represented by the fractional element networks are studied. Three sets of creep experimental data for polymer and rock are employed to demonstrate the effectiveness of these fractional derivative models. The corresponding constrained problem of nonlinear optimization is solved with an interior-point algorithm to obtain best fitting parameters of these fractional derivative models. The comparison results of measured values and calculated values versus time are displayed through graphics. The results demonstrate that the fractional Poynting–Thomson model is optimal in simulating the creep behavior of viscoelastic materials. And it also shows that the interior-point method is effective in the inverse problem to estimate parameters of fractional viscoelastic models.

1. Introduction

As one of the generalizations of the classical calculus, fractional calculus has been used widely in various fields of science and engineering. Quantity of mathematical knowledge on fractional integrals and derivatives has been acquired. Because of the long history dependence or the so-called memory effects, the fractional operator has been found to be a powerful tool for modeling the viscoelastic behaviors, particularly for building the time-dependent constitutive model [1–4].

The simplest fractional model of viscoelastic media, the fractional element (FE) model, as shown in Fig. 1(a), was proposed by Scott-Blair in 1947 [2] and the stress–strain relation has the form

$$\sigma(t) = \eta \frac{\mathrm{d}^{\alpha}\varepsilon(t)}{\mathrm{d}t^{\alpha}}, \quad \alpha \in (0, 1), \tag{1}$$

where α and η are the material-dependent constants. Here, $\mathrm{d}^{\alpha}/\mathrm{d}t^{\alpha}$ is the Riemann–Liouville fractional differential operator of order α. From a phenomenological view point, the fractional element model can be naturally generalized from springs and dashpots when the ordinary derivatives in their constitutive equations were replaced by the fractional ones. Indeed, it can be realized through hierarchical arrangements of springs and dashpots physically [1]. And, the physical meaning of the fractional order in Eq. (1) was explained as an index of memory [5]. Based on the above reasons, fractional viscoelastic models have successfully used in linear viscoelasticity recently. Replacing both the elastic element and the viscous element of the classical linear viscoelastic models [6] by the fractional elements, we have the fractional Maxwell (FM) model, fractional

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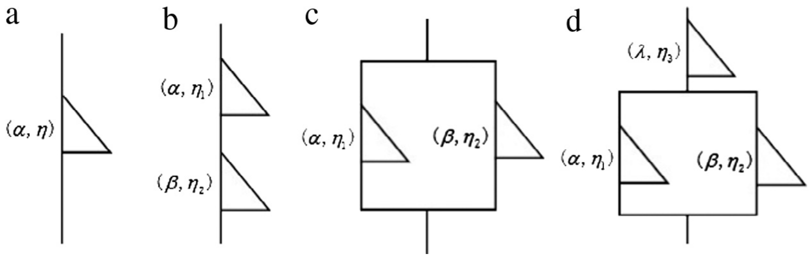

Fig. 1. Generalized viscoelastic models: (a) fractional element model; (b) fractional Maxwell model; (c) fractional Kelvin–Voigt model; (d) fractional Poynting–Thomson model.

Kelvin–Voigt (FKV) model and fractional Poynting–Thomson (FPT) model, as shown in Fig. 1(b)–(d). And a number of authors have used these fractional viscoelastic models to describe the properties of viscoelastic materials [1,2,7,8].

Now, fractional viscoelastic models are becoming more and more widespread due to their ability to describe the behavior of viscoelastic materials by using a small number of parameters [9]. It is critical to identity parameters of the models from experimental data. Song and Jiang [10] analyzed the characteristics of Sesbania gel and xanthan gum by using the fractional Jeffreys model and made confident predictions. Welch et al. [11] adopted different fractional models to fit the viscoelastic creep and stress relaxation of several polymeric materials. The relaxation modulus of PMMA and PTFE was measured and the experimental data can be appropriately simulated via a FM model [12]. A series of methods for identifying parameters of both the FKV model and the FM model are proposed by Lewandowski and Chorążyczewski [13]. And the validity and effectiveness of these procedures have been tested by using both artificially generated and experimental data. The technology of non-linear Levenberg–Marquardt method was also used in the inverse problems to estimate fractional model parameters in Ref. [14]. Arikoglu [15] proposed a new fractional derivative model, characterized by ten parameters, to demonstrate the viscoelastic behavior of polymeric damping materials and used genetic algorithms to identify the model parameters. More recently, Fan et al. [16] put forward a Bayesian method to estimate parameters in the fractional Zener model.

As an intermediate process and critical phenomenon, the constitutive relation of viscoelastic materials is one of the most successful application of the fractional operator theory [17]. And time-dependent behavior of viscoelastic materials, such as polymer, rock and mineral, is of great interest and fundamental importance for the widespread studies of the polymer, geological and geotechnical engineering [18,19]. A number of authors have worked on fractional derivative modeling. Among them we noticed that Zhou et al. [20] used a fractional derivative constitutive model to study the creep of salt rock; Celauro [21] proposed a fractional derivative model to describe the viscoelastic behavior of asphalt mixtures; Kang et al. [22] used a fractional non-linear creep model for coal; Wu et al. [23] proposed an improved Maxwell creep model on the basis of variable-order fractional derivatives to describe the mechanical property of salt rock. Fang et al. [24] utilized the FPT model to fit experimental data and found a significant improvement in the description of the strain recovery response of the shape memory polymers. More recently, Nadzharyan et al. [25] experimentally studied the behavior of magnetorheological elastomers in various magnetic fields and applied the FPT model to simulate its constitutive behavior. The results illustrated that the fractional rheological model can describe dynamic behavior of magnetorheological elastomers in magnetic field very well.

However, study on using constitutive model in the global mechanical behavior of a given material is far from enough [9]. Thus, we first review the linear fractional viscoelastic models and the corresponding creep strain functions in Section 2; In Section 3, based on the creep experimental data for polymers and rock, we carry out the identification of the fractional model parameters by using the interior-point algorithm to solve the corresponding constrained nonlinear optimization problem. Then we analyze the short and long time properties of the fractional viscoelastic models. Finally, some useful conclusions are given in Section 4.

2. Fractional viscoelastic models

Noting that stress is proportional to the zeroth derivative of strain for solids and to the first derivative of strain for fluids, it is natural to suppose that for viscoelastic materials stress should be proportional to strain of non-integer order, Eq. (1). This relationship has been given by Scott–Blair and is called the fractional element model by Schiessel et al. [1, Chap VII], which is denoted by a triangle or a ladder using two parameters (α, η) in Fig. 1(a). It should be noted that Xu and Tan [7] eliminated the restriction on the parameters of fractional element in Ref. [1], and α may be assigned with any values of the closed interval $[0, 1]$. With the inverse Laplace transform method, the creep strain function or the creep compliance of the FE model can be easily obtained

$$J(t) = \frac{t^{\alpha}}{\eta\,\Gamma(1+\alpha)}. \tag{2}$$

The Maxwell model, the Kelvin–Voigt model and the Poynting–Thomson model are the classical rheological constitutive equations [6]. Replacing the elastic and viscous elements in the above mechanical models by the fractional element, Schiessel

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