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A note on the Gaussons of some new logarithmic evolution equations

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ABSTRACT

In this paper, we study the Gaussian solitary waves for some nonlinear evolution equations with logarithmic nonlinearities. These studied logarithmic evolution equations are the generalized logarithmic BBM equations, the logarithmic (2+1)-dimensional KP-like equations, the logarithmic (3+1)-dimensional KP-like equations, the generalized logarithmic (2+1)-dimensional Klein-Gordon equations and the generalized logarithmic (3+1)-dimensional Klein-Gordon equations. We not only prove that they possess Gaussons: solitary wave solutions of Gaussian shape but also derive the relationships among the parameters.

1. Introduction

In the last years, researches of the exact solutions of nonlinear partial differential equations have been flourishing, because a lot of nonlinear complex physical phenomena arise in many research fields. For example, physics, theory of deep water waves, chaos and ecology, fluid dynamics, and so on. The study of the exact solutions can greatly help us understand the mechanism that governs nonlinear complex physical phenomena. Many research people have paid more attentions to this research field of significance [1–45]. A few excellent methods have been derived to find exact solutions of nonlinear partial differential equations. Some important methods are the homogeneous balance method, the hyperbolic function method, the F-expansion method and the variable-detached method, the Bäcklund transformation method, the Jacobi elliptic function method, the extended *tanh*-function method, and so on [1–11]. The aim of this research is to investigate the Gaussian solitary wave solutions of some nonlinear evolution equations: the generalized logarithmic BBM equations, the logarithmic (2 + 1)-dimensional KP-like equations, the logarithmic (3 + 1)-dimensional KP-like equations, the generalized logarithmic (2+1)-dimensional Klein–Gordon equations and the generalized logarithmic (3+1)-dimensional Klein–Gordon equations. We will show the existence of Gaussons of these studied logarithmic evolution equations, and derive the relations among the parameters.

Some nonlinear models, such as the Kortewegde Vries (KdV) equation, the Boussinesq equation, the BBM equations, the KP equations and the Klein–Gordon equations, and so on, are very important to the dynamics of shallow-water waves [15–31]. The BBM equations possess-like KdV-global solutions for very general initial data, in particular all physically relevant waves; the KP equations describe solitons in weakly dispersive media, particularly in fluid dynamics. In the meantime, a

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generalized KdV equation, called logarithmic KdV(log-KdV) [32], was derived as formal asymptotic limits of the FPU lattices defined by

$$v_t + (v \ln|v|)_x + v_{xxx} = 0, \tag{1}$$

which has Gaussian solitary wave solutions.

As a matter of fact, the logarithmic nonlinearities appear in many nonlinear equations such as the Schrödinger equation, KdV equation, Sine-Gordon equation, Boussinesq equation, and so on. A variety of nonlinear evolution equations with logarithmic nonlinearities have been studied [33–45]. In this work, we will study the generalized logarithmic BBM equation:

$$v_t + v_x + \frac{2\epsilon}{3}(v \ln|v|^n)_x + \delta^2(\alpha v_{xxx} + \beta v_{xxt}) = 0,$$

with $\beta \le 0$ and $\alpha = \frac{1}{6} + \beta$ to prevent ill-posedness.

It is noted that this equation is a new generalized logarithmic BBM equation and the Gaussian solitary wave solutions is given.

Then using the sense of the KP equation to establish the following logarithmic KP-like equations

$$\begin{aligned}
\left(v_t + v_x + \frac{2\epsilon}{3}(v \ln|v|^n)_x + \delta^2(\alpha v_{xxx} + \beta v_{xxt})\right)_x + \sigma v_{yy} &= 0,\\
(v_t + (v \ln|v|^n)_x + v_{xxx})_x + \sigma v_{yy} + \gamma v_{zz} &= 0,\\
(v_t + d v_{xx}(\ln|v|^n)_x - v_{xxx})_x + \sigma v_{yy} + \gamma v_{zz} &= 0,\\
\left(v_t + \frac{d v_x^2}{2v}(\ln|v|^n)_x - \frac{1}{2}v_{xxx}\right)_x + \sigma v_{yy} + \gamma v_{zz} &= 0,\\
\left(v_t + v_x + \frac{2\epsilon}{3}(v \ln|v|^n)_x + \delta^2(\alpha v_{xxx} + \beta v_{xxt})\right)_x + \sigma v_{yy} + \gamma v_{zz} &= 0.
\end{aligned}$$

In this paper, we introduce these aforementioned new (2 + 1)-dimensional and (3 + 1)-dimensional KP-like equations herein, and will obtain their Gaussian solitary wave solutions.

The generalized logarithmic Klein–Gordon equations are given as follows

$$\begin{aligned}
v_{tt} - \alpha^2 v_{xx} - \beta^2 v_{yy} + bv + dv \ln|v|^n &= 0,\\
v_{tt} - \alpha^2 v_{xx} - \beta^2 v_{yy} - \gamma^2 v_{zz} + bv + dv \ln|v|^n &= 0,
\end{aligned}$$

where α, β are free parameters.

This paper is organized as follows: we address the Gaussian solitary wave solutions of the generalized logarithmic BBM equations in Section 2, the logarithmic (2+1)-dimensional KP-like equation in Section 3, the logarithmic (3+1)-dimensional KP-like equations in Sections 4–7, and the generalized logarithmic (2 + 1)-dimensional and (3 + 1) Klein–Gordon equations in Sections 8 and 9, respectively. Finally, some discussions are given in Section 10.

2. The generalized logarithmic BBM equation

In this section, we will study the generalized logarithmic BBM equation written in the following form:

$$v_t + v_x + \frac{2\epsilon}{3}(v \ln|v|^n)_x + \delta^2(\alpha v_{xxx} + \beta v_{xxt}) = 0, \tag{2}$$

with $\beta \le 0$ and $\alpha = \frac{1}{6} + \beta$ to prevent ill-posedness. To obtain the Gaussons of this equation, we first assume that

$$v(x, t) = e^{u(x,t)}, \tag{3}$$

then Eq. (2) becomes

$$u_t + u_x + \frac{2n\epsilon}{3}(u_x u + u_x) + \delta^2(\alpha(u_x^3 + 3u_x u_{xx} + u_{xxx}) + \beta(u_t u_x^2 + u_t u_{xx} + u_x u_{xt} + u_{xxt})) = 0, \tag{4}$$

where $u(x, t)$ is given by

$$u(x, t) = l + m(kx - ct - a)^2. \tag{5}$$

In order to determine the constants α, β, we plug (5) into (4) and solve the resulting system with respect to l, m, then we obtain

$$\begin{aligned}
l &= \frac{kn\epsilon + 3c - 3k}{2kn\epsilon},\\
m &= -\frac{n\epsilon}{6\delta^2 k(\alpha k - \beta c)},
\end{aligned} \tag{6}$$