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A novel high-order functional based image registration model with inequality constraint[☆]

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ABSTRACT

In this paper, a novel variational image registration model using a second-order functional as regularizer is presented. The main motivation for the new model stems from the LLT model (see Lysaker, 2003). In order to avoid mesh folding, inequality constraint on the determinant of the Jacobian matrix J of the transformation is also proposed. Furthermore, a fast solver is provided for numerical implementation of registration model with inequality constraints. Numerical experiments are illustrated to show the good performance of our new model according to the registration quality.

1. Introduction

Image registration which is also called image matching is one of the most useful and fundamental tasks in imaging processing domain. It is often encountered in many fields such as astronomy, art, biology, chemistry, medical imaging and remote sensing and so on. For an overview of image registration methodology see [1–4]. Here we focus on deformable image registration in a variational framework.

Usually, a variational image registration model can be described by following form: given two images, one kept unchanged is called reference R and another kept transformed is called template image T. They can be viewed as compactly supported function, $R, T : \Omega \to V \subset \mathbb{R}_0^+$, where $\Omega \subset \mathbb{R}^d$ be a bounded convex domain and d denotes spatial dimension of the given images. Without loss of generality, here we focus on $d = 2$ throughout this paper, but it is readily extendable to $d = 3$ with some additional modifications. Let $\boldsymbol{x} = (x, y)^\top$, then $d_\Omega = d_x d_y$. The purpose of registration is to look for a transformation φ defined by

$$\varphi : \mathbb{R}^2 \to \mathbb{R}^2,$$

such that transformed template image $T_\varphi(\boldsymbol{x}) := T(\varphi(\boldsymbol{x}))$ is similar to R as much as possible. To be more intuitive to understand how a point in the transformed template $T(\varphi(\boldsymbol{x}))$ is moved away from its original position in T, we can split the transformation φ into two parts: the trivial identity part and displacement $\boldsymbol{u}$, $\boldsymbol{u} : \mathbb{R}^2 \to \mathbb{R}^2$, $\boldsymbol{u} : \boldsymbol{x} \mapsto \boldsymbol{u}(\boldsymbol{x}) = (u_1(\boldsymbol{x}), u_2(\boldsymbol{x}))^\top$, that is to say

$$\varphi(\boldsymbol{x}) = \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}),$$

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thus it is equivalent to find the transformation φ and the displacement $\boldsymbol{u}$. The transformed template image $T(\varphi(\boldsymbol{x})) = T(\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x}))$ can be denoted $T(\boldsymbol{u})$. The image intensities of R and T are assumed to be comparable (i.e. in a monomodal registration) throughout this paper. In summary, the desired displacement $\boldsymbol{u}$ is a minimizer of the following joint energy functional

$$\min_{\boldsymbol{u}} \{\mathcal{J}_\alpha[\boldsymbol{u}] = \mathcal{D}(\boldsymbol{u}) + \alpha \mathcal{S}(\boldsymbol{u})\}, \tag{1}$$

where

$$\mathcal{D}(\boldsymbol{u}) = \frac{1}{2}\int_\Omega (T((\boldsymbol{x}) + \boldsymbol{u}(\boldsymbol{x})) - R(\boldsymbol{x}))^2 d_\Omega \tag{2}$$

represents similarity measure which quantifies distance or similarity of transformed template image $T(\boldsymbol{u})$ and reference R and other choice is discussed in [2], $\mathcal{S}(\boldsymbol{u})$ is regularizer which rules out unreasonable solutions during registration process, and $\alpha > 0$ is a regularization parameter which balance similarity and regularity of displacement.

And non-surprisingly, different regularizer techniques can produce different registration model, and the choice of regularizer techniques is very crucial for the solution and its properties, more details see [2]. At present, a great number of regularization functionals have been proposed, such as first order derivatives-based on total variation-, diffusion- and elastic regularizer registration models and higher order derivatives-based on linear curvature, mean curvature and Gaussian curvature ones, we can refer to [2,5–11]. As is well known, it is easy to implement for low order regularizations while they are less effective than high order ones in producing smooth displacement fields which are important in some applications including medical imaging. Although some of them high order regularizations generate more satisfactory registration results, more computational time is required owing to complexity of their regularization functional. In addition, mesh folding has not been taken into account. Searching for a model suitable for large and smooth deformation field with low computing time and no mesh folding is still a challenge. In this paper, a novel variational image registration model with inequality constraint is proposed.

The outline of the paper is organized as follows. In Section 2, we propose a new second-order functional based image registration model with inequality constraint then discuss its numerical method using a combination of the multiplier method and Gauss–Newton scheme with Armijos Line Search for solving the new model and further to combine with a multilevel method to achieve fast convergence in Section 3. Some experimental results including comparisons are illustrated in Section 4. Finally, conclusions and future work are summarized in Section 5.

2. The proposed new image registration model

In [12], Lysaker, Lundervold and Tai (LLT) proposed a second-order regularizer which has proved to be rather robust in image denoising, however, it has not been studied thoroughly yet for the registration problem (1). In addition, motivated by the fact that TV regularizer is much weaker than diffusion one in producing smooth displacement fields in image registration, we propose a new regularizer functional given by

$$\mathcal{S}^{\text{new}}(\boldsymbol{u}) = \frac{1}{2}\sum_{l=1}^{2}\int_\Omega |D^2(u_l)|^2 d_\Omega \tag{3}$$

where $|D^2(u_l)| = \sqrt{((u_l)_{xx})^2 + ((u_l)_{xy})^2 + ((u_l)_{yx})^2 + ((u_l)_{yy})^2} = \sqrt{\triangledown(u_l)_x \cdot \triangledown(u_l)_x + \triangledown(u_l)_y \cdot \triangledown(u_l)_y}$ is a convex functional, here symbol $\cdot$ denotes the inner product of the vectors, then Eq. (1) takes the following form

$$\min_{\boldsymbol{u}}\left\{\mathcal{J}_\alpha[\boldsymbol{u}] = \frac{1}{2}\int_\Omega \big(T(\boldsymbol{u}) - R\big)^2 d_\Omega + \frac{\alpha}{2}\sum_{l=1}^{2}\int_\Omega (\triangledown u_{l x}\cdot\triangledown u_{l x} + \triangledown u_{l y}\cdot\triangledown u_{l y}) d_\Omega\right\}. \tag{4}$$

In order to avoid mesh folding, an inequality constraint on the determinant of the Jacobian matrix J of the transformation φ is imposed on the objective function (4). Thus, the new registration model has the following form:

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & \left\{\mathcal{J}_\alpha[\boldsymbol{u}] = \frac{1}{2}\int_\Omega (T(\boldsymbol{u}) - R)^2 d_\Omega + \frac{\alpha}{2}\sum_{l=1}^{2}\int_\Omega (\triangledown(u_l)_x\cdot\triangledown(u_l)_x + \triangledown(u_l)_y\cdot\triangledown(u_l)_y) d_\Omega\right\}, \\ \text{s.t.} \quad & \mathcal{F}(\boldsymbol{u}) > 0, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathcal{F}(\boldsymbol{u}) &= \det(J(\varphi(\boldsymbol{x}))) \\ &= \begin{vmatrix} 1+(u_1)_x & (u_1)_y \\ (u_2)_x & 1+(u_2)_y \end{vmatrix} \\ &= (1+(u_1)_x)(1+(u_2)_y) - (u_1)_y(u_2)_x. \end{aligned} \tag{6}$$

Our proposed new model has the following advantages. Firstly, the new regularizer is rotational invariant. Secondly, the new registration model with regularizer $\mathcal{S}^{\text{new}}(\boldsymbol{u})$ does not require additional affine linear pre-registration step, we can refer