ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A numerical study of iterative substructuring method for finite element analysis of high frequency electromagnetic fields

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Iterative substructuring method
Iterative methods
Minimal residual method
Complex symmetric systems

ABSTRACT

This paper describes iterative methods for the high frequency electromagnetic analysis using the finite element method of Maxwell equations including displacement current. The conjugate orthogonal conjugate gradient method has been widely used to solve a complex symmetric system. However, the conventional method suffers from oscillating convergence histories in large-scale analysis. In this paper, to solve large-scale complex symmetric systems arising from the formulation of the E method, an iterative substructuring method like the minimal residual method is presented, and the performance of the convergence of the method is evaluated by numerical results. As the result, the proposed method shows a stable convergence behavior and a fast convergence rate compared to other iterative methods.

1. Introduction

The electromagnetic field simulation in the range of several megahertz to several gigahertz has high demand for industrial and medical applications. The finite element method (FEM) with the formulation of the $\boldsymbol{E}$ method has been used to solve the vector wave equations in high frequency electromagnetic problems. However, the iterative method for solving such finite element equation is known for having bad convergence. By increasing problem size with increasingly complex shape, the convergence is deteriorating further. Therefore, solving large-scale problems efficiently on the parallel computer is a crucial issue, and both a robust convergence for increasing problem size and a scalable parallel efficiency is in great demand [1–3]. For the high frequency electromagnetic field analysis, the finite element formulation of the $\boldsymbol{E}$ method yields a large-scale complex symmetric linear system. To solve this systems, the conjugate orthogonal conjugate gradient (COCG) method [4] with the shifted incomplete Cholesky preconditioning has been widely used. However, for the FEM with arbitrary and complex unstructured mesh, the incomplete Cholesky preconditioner has difficulty obtaining the high performance of both parallel efficiency and convergence. Although the multigrid method is well known as the fast iterative method, it faces the difficult problem to solve a large-scale coarse problem in the large-scale analysis.

As an efficient parallel computing method for large-scale finite element analysis (FEA), we have been studying the iterative substructuring method. The iterative substructuring method form is known as the domain decomposition method

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http://dx.doi.org/10.1016/j.camwa.2016.04.028

(DDM) based on the iterative method [5]. The DDM is expected to obtain scalable parallel efficiency on the distributed memory parallel computers [6]. The DDM was applied to large-scale FEA of structural mechanics [7], heat transfer [8], and nonlinear magnetostatic problems with the magnetic vector potential $\boldsymbol{A}$ as an unknown function [9–11]. Furthermore, the DDM algorithm for the high frequency electromagnetic problems based on the formulation of the $\boldsymbol{E}$ method was developed [12], and successfully solved a 100 million complex degrees of freedom problem [13,14]. The DDM is also expected to get fast convergence by effective preconditioners such as the balancing domain decomposition (BDD) [15] and the balancing domain decomposition based on constraints (BDDC) [16]. The finite element tearing and interconnecting (FETI) method [17] and the dual-primal FETI (FETI-DP) method [18], which are the dual method of BDD and BDDC respectively, are also well-known DDM algorithm. However, the iterative methods for the high frequency electromagnetic problems are not fully established. Therefore, this paper focuses on the iterative methods for the DDM algorithm.

In the non-overlapping DDM, the whole analysis domain is decomposed into subdomains, and the problem to be solved is also decomposed into subdomain-interior (subdomain) problems and a subdomain-interface (interface) problem. The iterative substructuring method solves the interface problem using the iterative methods with solving subdomain problems, which means to perform FEA in each subdomain. In the high frequency electromagnetic field analysis with the finite element formulation of the $\boldsymbol{E}$ method, the interface problem and the subdomain problems are also complex symmetric. Hence, the COCG method can be used to solve the interface problem. However, since that formulation leads to the ill-conditioned problem, the COCG method shows oscillating residual norm histories, and suffers from very slow convergence in the large-scale analysis. On the other hand, the conjugate orthogonal conjugate residual (COCR) method, which extends the conjugate residual method for Hermitian linear systems to complex symmetric linear systems, is expected to obtain smoothed convergence behavior [19]. The iterative substructuring method based on the COCR method was applied to the large-scale FEA of the high frequency electromagnetic fields and improved convergence compared with the COCG method [20], however, its convergence behavior remains oscillating tendency.

In this paper, an iterative substructuring method based on a MINRES-like_CS method based on computational procedures of the minimal residual (MINRES) method [21] is presented, and the performance of the convergence of the method is evaluated by numerical results. The formulation of the high frequency electromagnetic problems is described in Section 2. The iterative substructuring method with the iterative methods is discussed in Section 3. Section 4 shows some numerical examples.

2. Finite element formulation

Vector wave equations. Let Ω be a domain with the boundary $\partial\Omega$. The vector wave equations which describe an electromagnetic field with single angular frequency ω are derived from Maxwell's equations containing the displacement current. The vector wave equations describing an electric field $\boldsymbol{E}$ are given by (1) and (2) using the current density $\boldsymbol{J}$ and the electric field $\boldsymbol{E}$, and assigning j as an imaginary unit

$$\mathrm{rot}\left(\frac{1}{\mu}\mathrm{rot}\boldsymbol{E}\right) - \omega^2\varepsilon\boldsymbol{E} = j\omega\boldsymbol{J} \quad \text{in } \Omega, \tag{1}$$

$$\boldsymbol{E}\times \mathrm{n} = 0 \quad \text{in } \partial\Omega, \tag{2}$$

$$\boldsymbol{J} = \sigma\hat{\boldsymbol{E}}. \tag{3}$$

In (1) and hereafter, rot is the infinitesimal rotation of a 3-dimensional vector field, and described as follows.

$$\mathrm{rot}\boldsymbol{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)e_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)e_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)e_z, \tag{4}$$

where $\boldsymbol{E} = (E_x, E_y, E_z)$ is a vector field and e_x, e_y, e_z are the unit vector for the x, y, z axes. Permittivity and permeability are given by $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$ respectively, where ε_0 denotes vacuum permittivity, ε_r relative permittivity, μ_0 vacuum permeability, and μ_r relative permeability. In this formulation, the permittivity becomes complex permittivity $\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0\varepsilon_r' + \sigma/j\omega$. The electric field $\hat{\boldsymbol{E}}$ on known points is substituted into (1) by (3), where the electrical conductivity is denoted as σ. By solving (1) with imposing the boundary condition of (2), we calculate the electric field $\boldsymbol{E}$. The magnetic field $\boldsymbol{H}$ is then calculated from the electric field $\boldsymbol{E}$ as post-processing using Faraday's law of induction, which is expressed by

$$\mathrm{rot}\boldsymbol{E} - j\omega\mu\boldsymbol{H} = 0. \tag{5}$$

Finite element discretization. Next, we describe the finite element discretization. Let us decompose Ω into an union of tetrahedra. $\boldsymbol{E}_h$ is an electric field approximated by the Nédélec elements [22,23], and $\boldsymbol{J}_h$ is an electric current density approximated by the conventional piecewise linear tetrahedral elements. As a result, we have the finite element approximation

$$\iiint_\Omega \mathrm{rot}\boldsymbol{E}_h \cdot \frac{1}{\mu}\mathrm{rot}\boldsymbol{E}_h^* dv - \omega^2 \iiint_\Omega \varepsilon\boldsymbol{E}_h\cdot\boldsymbol{E}_h^* dv = j\omega\iiint_\Omega \boldsymbol{J}_h\cdot\boldsymbol{E}_h^* dv, \tag{6}$$

where $\boldsymbol{E}_h^* \times \mathrm{n} = 0$ on $\partial\Omega$.