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Truncated Hierarchical Loop Subdivision Surfaces and application in isogeometric analysis

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ABSTRACT

Subdivision Surface provides an efficient way to represent free-form surfaces with arbitrary topology. Loop subdivision is a subdivision scheme for triangular meshes, which is C^2 continuous except at a finite number of extraordinary vertices with G^1 continuous. In this paper we propose the Truncated Hierarchical Loop Subdivision Surface (THLSS), which generalizes truncated hierarchical B-splines to arbitrary topological triangular meshes. THLSS basis functions are linearly independent, form a partition of unity, and are locally refinable. THLSS also preserves the geometry during adaptive h-refinement and thus inherits the surface continuity of Loop subdivision surface. Adaptive isogeometric analysis is performed with the THLSS basis functions on several complex models with extraordinary vertices to show the potential application of THLSS.

1. Introduction

Isogeometric analysis (IGA) was originally introduced by Hughes et al. [1] and described in detail in [2]. With IGA, traditional design-through-analysis procedures such as geometry clean-up, defeaturing, and mesh generation are simplified or eliminated entirely. Additionally, the higher-order smoothness provides substantial gains to analysis in terms of accuracy and robustness of finite element solutions [3–5]. However, a global geometric discretization, based on NURBS, is usually not suitable as a basis for analysis. Many different methods have been developed in these years to define locally refinable splines, such as the (Truncated) Hierarchical B-splines [6–8], the (Analysis-suitable) T-splines [9–12], the PHT-splines [13–15], the LR B-splines [16,17] and the Modified T-splines [18]. Truncated hierarchical Catmull–Clark subdivision (THCCS) [19,20] generalized Truncated Hierarchical B-splines [8] to control grids of arbitrary topology. THCCS provide a method to define locally refinable splines on quadrilateral meshes with extraordinary nodes.

Recently locally refinable splines on triangular partitions also attract researchers's interest because of the flexibility and the popular using in classical finite element analysis of triangular partitions. Hierarchical bivariate splines on regular (type-I and type-II) triangular partitions were introduced in [21] and applied to numerical solving PDEs. Later, Jüttler et al. [22] generalized the truncated hierarchical B-splines [8] to hierarchies of spaces that are spanned by generating systems that potentially possess linear dependencies, a special box splines defined on criss-cross grid called Zwart–Powell (ZP) elements was discussed as an example. Speleers et al. [23,24] proposed hierarchical Powell–Sabin splines for isogeometric analysis applications, where Powell–Sabin splines are C^1 piecewise quadratic polynomials defined on a special refinement of any given triangulation.

Loop subdivision [25] is a subdivision scheme for triangular meshes. The limit surface defined by Loop subdivision is C^2 continuous except at a finite number of extraordinary vertices (an extraordinary vertex has other than six faces adjacent

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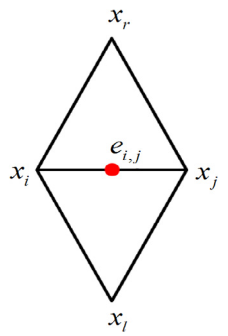
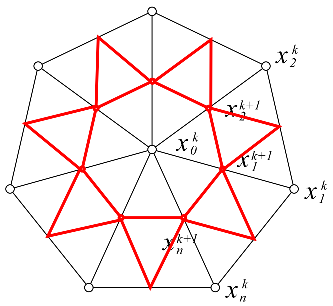


(a) Edge point rule.



(b) Vertex point rule.

Fig. 1. Mask for Loop subdivision.

to it) where the surface is G^1 continuous. Explicit Loop basis functions were explored by J. Stam [26] and have several nice properties: linear independence, partition of unity and local support. There recently have been a few works on the application of Loop subdivision in isogeometric analysis. Loop subdivision surfaces were used for describing the geometry of shell and the displacement fields in thin-shell finite element analysis [27]. Extended Loop subdivision surfaces were used in isogeometric analysis in [28], where Poisson equations with the Dirichlet boundary condition were considered and the approximation properties of extended Loop basis functions were established.

In this paper we introduce the truncation hierarchical mechanism [8] into Loop subdivision surfaces, which are called Truncated Hierarchical Loop Subdivision Surfaces (THLSS), to be adapted to triangular meshes with arbitrary topology and support local refinement. THLSS preserve the exact geometry when adaptive h-refinement is performed and inherit the surface continuity of Loop subdivision surfaces. THLSS basis functions are global linearly independent, form a partition of unity and have local support. We applied THLSS basis functions in isogeometric analysis on several complex geometries. The simulation results show potential wide application of the proposed method in integrating design and analysis. Through a benchmark numerical experiment, we demonstrate its efficiency with the comparison to the classical finite element analysis piecewise linear elements.

The paper is organized as follows: Section 2 briefly reviews Loop subdivision scheme including Stam's explicit basis functions. Section 3 presents the detail of THLSS construction. Section 4 shows several numerical experiments with the comparison to the FEA with linear elements and Loop basis functions. Section 5 is the conclusion and future work.

2. Loop subdivision surface

In this section, we briefly review Loop's subdivision scheme and the explicit basis functions introduced by J. Stam [26].

2.1. Loop subdivision scheme

Loop subdivision scheme is an approximating subdivision scheme. Referring to Fig. 1, let x_l and x_r be the two wing neighbor vertices of edge$[x_i x_j]$, then the new edge point added on this edge is defined as

$$e_{i,j} = \frac{3}{8}x_i + \frac{3}{8}x_j + \frac{1}{8}x_l + \frac{1}{8}x_r.$$

And for a vertex x_0^k at level k with neighboring vertices $x_i^k, i = 1, 2, \ldots, n$, where n is the valence of vertex x_0^k. The old vertex is updated to x_0^{k+1} according to

$$x_0^{k+1} = (1 - n\alpha)x_0^k + \alpha(x_1^k + x_2^k + \cdots + x_n^k),$$

where $\alpha = \frac{1}{n}[\frac{5}{8} - (\frac{3}{8} + \frac{1}{4}\cos(\frac{2\pi}{n}))^2]$. This linear relationship can be expressed by a so-called subdivision matrix. The repeated global refinement generates a sequence of meshes $\mathbb{M}^0, \ldots, \mathbb{M}^n$, where $\mathbb{M}^0$ is the initial control grid, and n is the number of subdivisions. As n goes to infinity, $\mathbb{M}^n$ converges to a limit surface. We call this limit surface as *Loop subdivision surface*.

2.2. Loop basis functions

An alternative way to obtain the limit surface takes advantage of the Stam's basis functions [26]. These basis functions are analogous to B-spline basis functions, whereas each mesh $\mathbb{M}^l$ is served as a control grid. Thus we can express the limit surface S_{limit} by a mapping from the parametric domain to the physical domain,

$$S_{limit}(v, w) = \sum_{i=1}^{N^l} B_i^l(v, w)P_i^l, \quad (1)$$