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Boundedness and decay behavior in a higher-dimensional quasilinear chemotaxis system with nonlinear logistic source

Jiashan Zheng [a,*], Yifu Wang [b]

[a] School of Mathematics and Statistics Science, Ludong University, Yantai 264025, PR China
[b] School of Mathematics and Statistics, Beijing Institute of Technology, Beijing 100081, PR China

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ABSTRACT

This paper is concerned with a class of quasilinear chemotaxis systems generalizing the prototype

$$\begin{cases} u_t = \Delta u^m - \nabla\cdot(u\nabla v) + \mu u - u^r, & x\in\Omega, t>0,\\ v_t = \Delta v - v + u, & x\in\Omega, t>0,\\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x\in\partial\Omega, t>0,\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x) & x\in\Omega, \end{cases} \tag{0.1}$$

in a smooth bounded domain $\Omega\subset\mathbb{R}^N (N\geq 2)$ with parameters $m, r\geq 1$ and $\mu\geq 0$. The PDE system in (0.1) is used in mathematical biology to model the mechanism of chemotaxis, that is, the movement of cells in response to the presence of a chemical signal substance which is in homogeneously distributed in space. It is shown that if

$$m\begin{cases} > 2-\dfrac{2}{N} & \text{if } 1<r<\dfrac{N+2}{N},\\ > 1+\dfrac{(N+2-2r)^+}{N+2} & \text{if } \dfrac{N+2}{2}\geq r\geq \dfrac{N+2}{N},\\ \geq 1 & \text{if } r>\dfrac{N+2}{2}, \end{cases}$$

and the nonnegative initial data $(u_0, v_0)\in C^{\iota}(\bar{\Omega})\times W^{1,\infty}(\Omega)(\iota>0)$, then (0.1) possesses at least one global bounded weak solution. Apart from this, it is proved that if $\mu=0$ then both $u(\cdot,t)$ and $v(\cdot,t)$ decay to zero with respect to the norm in $L^{\infty}(\Omega)$ as $t\to\infty$.

* Corresponding author.
E-mail address: zhengjiashan2008@163.com (J. Zheng).

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1. Introduction

In this paper, we consider the Neumann initial–boundary value problem for a full chemotaxis system with generalized volume-filling effect and nonlinear logistic source

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(S(u)\nabla v) + \mu u - u^r, & x\in\Omega, t>0,\\ v_t = \Delta v + u - v, \quad x\in\Omega, t>0,\\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, \quad x\in\partial\Omega, t>0,\\ u(x,0)=u_0(x), \qquad v(x,0)=v_0(x), \ x\in\Omega \end{cases} \tag{1.1}$$

in a bounded domain $\Omega\subset\mathbb{R}^N (N\geq 2)$ with smooth outward normal vector field ν on $\partial\Omega$, where $S(u)=u$ is the sensitivity function, parameter $\chi>0$, and the initial data $u_0\in C^{\iota}(\overline{\Omega})(\iota>0)$ and $v_0\in W^{1,\infty}(\Omega)$ are assumed to satisfy $u_0, v_0\geq 0$ in $\overline{\Omega}$.

The problem of this type is used to describe theoretically the process of chemotaxis, the biological phenomenon of the oriented movement of cells or organisms in response to a chemical signal. The pioneering works of chemotaxis model were introduced by Patlak [1] in 1953 and Keller and Segel [2] in 1970, and we refer the reader to the survey by Hillen and Painter [3] and Horstmann [4], where a comprehensive information of further examples illustrating the outstanding biological relevance of chemotaxis can be found. In this paper, we assume that the diffusion function $D(u)$ satisfies

$$D\in C^{\theta}_{loc}([0,\infty)) \quad \text{for some } \theta>0, \tag{1.2}$$

and there exist some constants $m\geq 1$ and $C_D>0$ such that

$$D(u)\geq C_D u^{m-1} \quad \text{for all } u\geq 0. \tag{1.3}$$

During the past decades, the variants of (1.1) without cell kinetics have been studied extensively by researchers and the main issue of the investigation was whether the solutions of the models are bounded or blow-up (see e.g. Horstmann and Winkler [5], Tao and Winkler [6], Ishida et al. [7], Winkler [8]). In many applications, blow-up phenomena do not appropriately reflect the respective experimentally observable behavior. Accordingly, considerable efforts have been devoted to developing models in which blow-up phenomena are prevented. In this context it is recognized that cell proliferation terms of logistic type, such as contained in (1.1), form the possibly simplest among the blow-up preventing mechanisms. In fact, the presence of such logistic terms is sufficient to suppress any blow-up in many relevant situations. For example, if $r=2$ (quadratic type), Winkler [9] discussed the global boundedness of classical solutions to problem (1.1) on a smooth bounded convex domain under the assumption that either $N\leq 2$, or the logistic damping effect is large enough. A large number of facts indicate that Keller–Segel-growth systems with logistic-type kinetics of superlinear, not necessarily quadratic type. In fact, when $D(u)=(u+1)^{-\alpha}$, the sensitivity function $S(u)=u$ is replaced by $S(u)=(u+1)^{\beta-1}$ with $0<\alpha+\beta<\frac{2}{N}$ and the logistic term f satisfies

$$f(u)\leq \mu - bu^r \quad \text{for all } u\geq 0 \tag{1.4}$$

with some $\mu\geq 0$, $b>0$ and $r=2$, Wang et al. [10] obtained the unique global uniformly bounded classical solution (u,v) of problem (1.1), but there is not any available result on the boundedness of the solution when $\alpha+\beta>\frac{2}{N}$. Furthermore, assuming that the logistic source $f\in C^{\infty}([0,\infty))$ satisfies

$$f(u)\leq \mu u - bu^2 \quad \text{for all } u\geq 0 \tag{1.5}$$

and the diffusion function D and the sensitivity function S fulfill

$$\begin{aligned} &D,S\in C^2([0,\infty)) \quad \text{and} \quad D(s)\geq 0 \text{ for all } s\geq 0,\\ &c_1 s^p \leq D(s) \quad \text{for all } s\geq s_0,\\ &c_1 s^{\beta}\leq S(s)\leq c_2 s^{\beta} \quad \text{for all } s\geq s_0 \end{aligned}$$

with $c_2>c_1>0$, $s_0>1$ and $p,\beta\in\mathbb{R}$, Cao [11] proved that if $\beta<1$, then the classical solution of (1.1) is globally bounded, whereas the question whether or not blow-up may occur is left therein when $\beta>1$. Recently, in [12], it is proved that if $0<\alpha+\beta<\max\{r-1+\alpha,\frac{2}{N}\}$ or b is big enough when $\beta=r-1$, then the classical solutions to the corresponding system are uniformly bounded. There is also other alternative as the degenerate sensitivity function, degenerate problems have been studied to prevent blow up (see Chamoun et al. [13], Lorz [14]). Going beyond these boundedness statements, a number of results are available which show that the interplay of chemotactic cross-diffusion and cell kinetics of logistic-type may lead to quite a colorful dynamics. For instance, if $D(u)\equiv 1$, $S(u)=u$, $f(u)=u-bu^2$ and the ratio $\frac{b}{\chi}$ is sufficiently large, Winkler [15] proved that the unique nontrivial spatially homogeneous equilibrium given by $u=v\equiv\frac{1}{b}$ is globally asymptotically stable in the sense that for any choice of suitably regular nonnegative initial data (u_0,v_0) such that $u_0\not\equiv 0$, the problem (1.1) possesses a uniquely determined global classical solution (u,v) with $(u,v)|_{t=0}=(u_0,v_0)$ which satisfies

$$\left\|u(\cdot,t)-\frac{1}{b}\right\|_{L^{\infty}(\Omega)}\to 0 \quad \text{and} \quad \left\|v(\cdot,t)-\frac{1}{b}\right\|_{L^{\infty}(\Omega)}\to 0$$