ARTICLE IN PRESS
Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Spectrally accurate algorithm for analysis of convection in corrugated conduits

A. Abtahi *, M.Z. Hossain, J.M. Floryan

Department of Mechanical and Materials Engineering, The University of Western Ontario, London, Ontario, N6A 5B9, Canada

ARTICLE INFO

Article history:
Received 22 March 2016
Received in revised form 3 August 2016
Accepted 17 September 2016
Available online xxxx

Keywords:
Spectral discretization
Convection
Immersed boundary conditions method
Linear solvers

ABSTRACT

An algorithm suitable for analysis of convection problems in corrugated conduits has been developed. The algorithm uses the immersed boundary conditions (IBC) concept to capture the effects of corrugated walls. The field equations are discretized on a regular domain surrounding the flow domain using Fourier expansions in the stream-wise direction and Chebyshev expansions in the wall-normal direction. The boundary conditions are expressed in the form of constraints and the spectrally accurate discretization of these constraints is discussed. The solution is carried out using the fix flow rate constraint. Discretization as well as a method for the direct inclusion of this constraint in the flow solver is presented. Several tests are used to demonstrate the spectral accuracy of the solution. The solution process relies on iterations with an efficient solver taking advantage of the structure of the coefficient matrix used to solve the linear system. The limitations of the algorithm are discussed and gains associated with the over-determined formulation are presented.

1. Introduction

Convective heat transfer occurs in many applications and, thus, determining its characteristics is of utmost importance. The fundamental aspects of convection have been studied in idealized geometries for many decades, e.g. natural convection between smooth parallel plates, using both theoretical and experimental approaches [1]. There are numerous case studies focused on the applied aspects of convection and involving specialized geometries [2–8]. There is, however, a gap between the fundamental and applied studies, and its elimination would yield practically important information about convection in more complex geometries. One of the areas where the fundamental information is missing is the effect of surface roughness/corrugations on natural and mixed convection, and on the onset of secondary states in buoyancy driven motions. This particular issue becomes prominent in micro-conduits where the roughness size cannot be reduced to a negligible level using currently available manufacturing techniques [9–15].

Analysis of the effects of roughness on heat transfer processes must address the issue of modelling of the roughness geometry. As there is an uncountable number of possible shapes, it appears that a general solution is not possible. Typical approaches rely on the selection of the geometry of interest and construction of boundary fitted coordinates followed by a solution of the field equations [9,16–18]. The prediction of the onset of secondary states requires a highly accurate solution of the stationary state, but very few high-accuracy grid generation methods are available, e.g. conformal mapping based on the Schwartz–Christoffel transformation whose parameters can be determined with near spectral accuracy [19–21]. One alternative is offered by the domain transformation method, which relies on analytically mapping the irregular physical domain

* Corresponding author.
E-mail address: sabtahi4@uwo.ca (A. Abtahi).

http://dx.doi.org/10.1016/j.camwa.2016.09.019

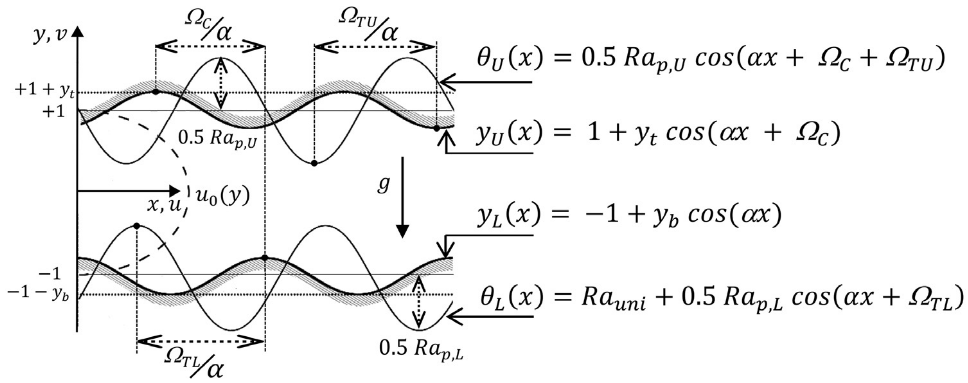

Fig. 1. Schematic diagram of the flow system.

to a rectangular computational domain. However, such mappings are available only for a limited class of geometries [22,23]. The other alternative involves the immersed (or fictitious) boundaries concept, whose origin can be traced to the analysis of moving boundary problems and the development of interface tracking methods [24]. In this class of problems, the grid is fixed while the solution domain travels through this grid. The available algorithms rely either on surface or on volume tracking, e.g. VOF (Volume of Fluid), MAC (Marker and Cell) and Level Set methods [25–30]. The standard form of these algorithms relies on low-order spatial accuracy consistent with the diffused boundary locations resulting from the tracking procedures.

The immersed boundaries concept has re-emerged recently, resulting in its rapid development [31,32]. The common limitation is the spatial accuracy, as most of these methods are based on either low-order finite-difference, finite-volume or finite-element techniques [31–35]. The second, less known limitation is the use of the local fictitious forces required to enforce the no-slip and no-penetration conditions. These forces locally affect the flow physics and this may lead to incorrect estimates of derivatives of flow quantities, i.e. misrepresentation of the local wall shear. This problem is likely to be more pronounced in the case of methods with high spatial accuracy.

The spectrally-accurate version of the immersed boundary concept, referred to as the Immersed Boundary Conditions (IBC) method, was proposed in 1999 [36] for the solution of the Navier–Stokes equations and is still the only method which guarantees spectral accuracy of the complete solution rather than just the spectral discretization of the field equations. This method has been extended to two- and three-dimensional conduction problems [37,38] as well to conduction problems with moving boundaries [39].

This work is focused on the extension of the IBC algorithm to non-isothermal conditions. Since the main goal of the analysis is to develop techniques for the analysis of roughness effects, we shall invoke the reduced geometry model in which the geometry is projected onto a convenient reference functional space and elements of this space relevant for hydrodynamics are sought. This leads to the analysis of sinusoidal grooves which represent the dominant terms in the geometry projection onto the Fourier space. It has been demonstrated that this approach provides a general answer to the issue of roughness effects with accuracy sufficient for most applications. Section 2 provides a description of the model problems used to illustrate the algorithm. Section 3 discusses the discretization, with Section 3.1 focused on the discretization of the field equations, Section 3.2 describing the discretization of the boundary conditions and Section 3.3 discussing the discretization of the flow rate constraint. Section 4 describes the solution process, Section 5 is focused on the linear solvers and Section 6 discusses the evaluation of the pressure field. Section 7 discusses the performance of the algorithm while Section 8 describes the gains associated with the overdetermined formulation. Section 9 provides a short summary of the main conclusions.

2. Problem Formulation

Consider channel formed by two horizontal corrugated plates (see Fig. 1) whose geometries have the form

$$y_L^*\left(x^*\right) = -h^* + C_L^* \cos\left(\alpha^* x^*\right), \tag{2.1a}$$

$$y_U^*\left(x^*\right) = h^* + C_U^* \cos(\alpha^* x^* + \Omega_C^*), \tag{2.1b}$$

where the subscripts L and U refer to the lower and upper plate, respectively, C_L^* and C_U^* are the amplitudes of the corrugations at the lower and upper plates, respectively, Ω_C^* stands for the phase shift between them, α^* is their wave number and stars denote dimensional quantities. The channel is periodic with the wavelength $\lambda^* = 2\pi/\alpha^*$ and extends to $\pm\infty$ in the x-direction with the mean distance between the plates $2h^*$. The gravitational acceleration g^* is acting in the negative y-direction. The steady, incompressible flow of a Newtonian fluid is driven in the positive x-direction by an externally imposed pressure gradient. The fluid has thermal conductivity k^*, specific heat c^*, thermal diffusivity $\kappa^* = k^*/\rho^* c^*$, kinematic viscosity ν^*, dynamic viscosity μ^*, thermal expansion coefficient Γ^* and variations of the density ρ^* follow the Boussinesq approximation. All material properties are evaluated at the reference temperature.