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A constrained assumed modes method for solution of a new dynamic equation for an axially moving beam

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ABSTRACT

Flexible beams with prismatic joints have complicated differential equations. This complexity is mostly due to axial motion of the beam. In the present research, a horizontal flexible link sliding through a passive prismatic joint while attached to a rigid link of a robot moving in a vertical direction is considered. A body coordinate system is used which aids in obtaining a new and rather simple form of the differential equation without the loss of generality. To model the passive prismatic joint, the motion differential equation is written in a form of virtual displacement. Next, a solution method is presented for the lateral vibrations of the beam referred to as "constrained assumed modes method". Unlike the traditional assumed modes method, in the proposed constrained assumed modes method, the assumed mode shapes do not each satisfy the geometrical boundary conditions of the point where passive prismatic joint is located. Instead, by writing additional constraint equations the combination of the assumed modes will satisfy the geometrical boundary conditions at location of the passive prismatic joint. Two case studies for the effect of axial motion on lateral vibration of the beam are presented. Approximate analytical results are compared with FEM results.

1. Introduction

Many researchers have studied the motion of a flexible beam with prismatic joint. As a beam slides through a fixed prismatic joint, lengths of two parts of the beam on the two sides of the prismatic joint vary with time. Consequently, time-variant or space-variant boundary conditions must be considered at two free ends of the beam. Additionally, due to axial motion of the beam, total time derivative of lateral displacement for a beam element is not equal to its partial time derivative. Therefore, more terms are added to differential equation of the beam to include the effect of velocity and acceleration of its axial motion on the lateral vibration. These are some of the reasons which make this problem challenging.

In Ref. [1] which can be accounted as the first complete modeling of an axially moving beam, several nonlinear partial differential equations and algebraic equations are obtained. The solving of these equations is performed by applying the several simplifications. Ref. [2] presented modeling of a flexible beam extruding from a rotating rigid body. They modeled the flexible beam as a series of elastically connected rigid links. Ref. [3] converted the partial differential equation of motion into a set of ordinary differential equation and solved it. Ref. [4] derived the governing motion equation using energy method. They used time-variant boundary conditions and time-variant mode shapes. Refs. [5,6] investigated dynamic modeling and

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feedback control of a flexible link with a concentrated mass at one end. Their solution method separately obtains beam vibration at each side of the prismatic joint. By means of a concept of group velocity in Ref. [7], a non-dimensional form is presented for the Euler–Bernoulli beam equation which leads to the separation of variables and the solution by using the assumed modes method. Considering the existing body of the literature, one finds that when beam length increases, the amplitude of vibrations increases which results in a destabilizing effect. Conversely, when beam length decreases, the amplitude of vibrations decreases which results in a stabilizing effect. Ref. [8] applied more exact elasticity theories for the modeling of a vertically and axially flexible beam with tip concentrated mass and derived nonlinear differential equations of motion and time-variant boundary conditions. However, solution for the differential equation was not presented. Ref. [9] used the same problem as used by Refs. [5,6] and modeled lateral vibrations of the beam on the two sides of the prismatic joint. For their modeling, they used two coordinate systems for the two sides of the prismatic joint. Hamilton's Principle was used for driving differential equations. Ref. [10] performed a stability analysis of an axially moving beam. The majority of the above references used the assumed modes method and obtained solution of the motion equation for one side of the prismatic joint. Additionally, boundary conditions at the prismatic joint and the free end of the beam are assumed to be clamped and time-variant, respectively.

Refs. [11,12] presented a dynamic model and simulation of a translating beam with rotary joint. They conclude that number of the assumed modes used significantly affects the beam solution. Ref. [13] developed three-dimensional modeling of a flexible link with prismatic joint and considered axial, lateral and torsional vibrations. They also obtained an approximate solution for lateral vibration using the perturbation method. Many researchers have used finite element method, FEM, for solving the beam with prismatic joint, Refs. [14–16]. Ref. [16] used Rayleigh's theory for beam element. Refs. [17,18] focused on a deploying/extruding flexible beam emerged in a fluid and used numerical methods to obtain solution of the derived motion equations. Ref. [19] presented mathematical modeling of a flexible link having rotating-prismatic joint with a concentrated tip mass.

Several constraint formulations [20,21] can be used in order to model the kinematic joints. Ref. [20] applied the kinematic constraints of the joints using the modal coordinates. However, in more recent researches [21,22], nodal coordinates (FEM) are preferred. The floating frame of reference formulation [22] and the absolute nodal coordinate formulation [21] are two mostly used methods in the FEM models. Type of the selected coordinates can create nonlinearity or other complexities in the system of equations. For example, in the modeling of the prismatic joint, interference of a global coordinate system with a body coordinate system create geometric nonlinearities, see [21]. Number of the modal coordinates is usually less than the number of nodal coordinates, and therefore, the size of the system of equations is reduced. However, it seems that assumed modes method is less employed than the finite element method in the modeling of the translational joints such as the prismatic joints.

Considering the available body of literature, to the best of authors' knowledge, a dynamic equation, which simultaneously provide vibration response for both sides of a beam sliding through a prismatic joint has not been presented. Additionally, the analytical dynamic models presented for the desired problem do not provide the reaction forces at the prismatic joint. Finally, it seems that the prismatic joint constraints are not used in mesh-free methods such as assumed modes method. Authors of the present paper previously performed dynamic and vibration of $\underline{P}$R-$\underline{P}$RP, 3-$\underline{P}$RP and 3-$\underline{P}$SP parallel robots in which flexible moving platforms have prismatic joints [23–25].

In this research, by means of a body coordinate system and without the loss of generality, a new and simpler form of motion equation for an axially moving beam sliding through a prismatic joint is obtained. The prismatic joint is also assumed to have a motion in vertical direction. Next, a solution method is presented for the motion equation of the beam, which is referred to as "constrained assumed modes method". This method applies the prismatic joint constraints on the assumed modes method. In the assumed modes method, each of the assumed mode shapes must satisfy all the geometrical boundary conditions. However, using the presented method, the assumed mode shapes do not each satisfy the geometrical boundary conditions of the point where prismatic joint is located. Instead, by writing additional constraint equations at location of the prismatic joint, the combination of the assumed modes will satisfy the geometrical boundary conditions at location of the prismatic joint. The proposed solution method has not been used for this type of problems before. The proposed solution method obtains the constraint forces and allows direct dynamic and vibration of the beam to be solved simultaneously. Finally note that it is the intent of this paper to extend its finding to more complex robotic systems. Therefore, as vibration with high amplitude is not desired in an industrial system then this paper focuses on a system with low amplitude vibrations.

2. Dynamical model

Consider a two-link robot with a rigid first link sliding through an active prismatic joint and a long flexible second link sliding through a passive prismatic joint as shown in Fig. 1.

Axial motion of the flexible link is in the x-direction and its lateral vibration is in the z-direction. The flexible link is considered as a free–free beam with a passive prismatic joint along the beam. The passive prismatic joint is attached to end of the rigid first link and therefore has the same motion as the rigid first link. It is assumed that there is no friction between the prismatic joint and the flexible link. In addition, a horizontal driving force is applied on the left end of the flexible link to provide its axial motion. It is also assumed that the flexible link has zero initial conditions. Additionally, to focus on the vibration behavior of the flexible link, the mass of the first link is assumed to be zero. An element of the beam is considered