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Hybrid central-upwind finite volume schemes for solving the Euler and Navier–Stokes equations

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ABSTRACT

A novel method based on the hybridization of central schemes and upwind schemes is proposed for finite volume discretization of the Euler and Navier–Stokes equations on multi-blocked structured grids. The developed methodology takes the advantages of the low diffusion characteristic from an improved central scheme and the discontinuity-capturing capability from a characteristic-based upwind scheme. A simple and efficient improvement to the scalar dissipation and matrix dissipation model for the central scheme is suggested to enhance the accuracy of the existing methods. The resulting hybrid schemes are as compact as the underlying finite volume methods and therefore easy to implement. Numerical results for a wide range of flow conditions demonstrate the method simultaneously obtains the desired accuracy and sharp, oscillation-free shock transition for both the inviscid and viscous simulation.

1. Introduction

For a long time the numerical discrete methods in computational fluid dynamics have been developed based on two quite different standpoints. One standpoint considers that the discrete zone consists of discontinuous solutions and the physics of wave propagation is accounted for by solving the Riemann problems at element interfaces. Typical techniques of such approaches include Flux Difference Splitting (FDS) schemes [1], Flux Vector Splitting (FVS) schemes [2], and AUSM-type schemes [3], and these methods can be classified as the upwind schemes. On the contrary, the other standpoint considers that the discrete zone consists of smooth solutions and the discontinuity is treated as continuous solutions with large gradient. The methods developed upon this standpoint can be classified as central schemes and the most typical techniques of such approaches are artificial viscosity methods.

The upwind schemes have won high praises because they can capture shock and contact discontinuity with high resolutions. The prototype of upwind schemes is the original first-order Godunov scheme and as a sequel to Godunov's method, MUSCL (monotone upstream scheme for conservation law) method [4] increases the formal order of the scheme while preserving its stability. Due to the ability to preserve stability, monotonicity as well as greater formal order of accuracy, MUSCL methods have become a widely used standard today. For flows containing strong shock waves or contact discontinuities, the use of limiters becomes necessary for preventing unphysical oscillations that may compromise solutions' accuracy and stability. However, it is well-known that no classical limiter has been found to work well for all problems. Another drawback of such slope limiters is to avoid the total convergence of numerical method due to the fact that even machine-order differences between neighbor cells may activate the limitation process.

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The prototype of central schemes is the first-order Lax–Friedrichs scheme and today the most common approach of central schemes is originated from the seminal work of Jameson et al. [5]. In this method numerical dissipation models are a mixture of the second-difference and the fourth-difference terms. By using ideas from characteristics-based upwind scheme, matrix dissipation models developed in [6] have offered comparable accuracy as high-resolution upwind scheme. The advantage of the matrix-dissipation scheme is that it bypasses the need for gradient reconstruction and all the problems involved in that procedure. However the central schemes are known to have strong dependencies on problem-dependent parameters which are determined mostly according to the experience of the users. Also the inexact correspondence to an approximate Riemann flux function makes the method questionable for high-speed flows [7]. Thus, these central schemes' applications are limited.

For general fluid dynamic computations, the numerical scheme should be designed to have high accuracy in smooth regions of the flow field and high resolution on the shock waves and contact discontinuities. However as two most widely-used numerical discrete methods, upwind schemes and central schemes seem to both have their own advantages and meantime fail to overcome their intrinsic defects. A natural idea is then to combine the advantages and meantime avoid the disadvantages of two schemes. Such so-called hybrid approaches [8–10] have been developed in the framework of the finite difference discretization, in which a high order compact or linear scheme is used in the smooth flow regions and a shock capturing scheme is used in the vicinity of flow discontinuities. In the framework of the finite volume (FV) discretization, hybrid schemes have been constructed in the literature [11,12]. The method is to utilize compact or upwind reconstructed schemes in smooth regions and then apply the shock-capturing schemes to handle the discontinuities. A class of compact-reconstruction weighted essentially non-oscillatory (CRWENO) scheme is recently proposed in [13]. The main idea of the scheme is to combine lower-order compact stencils to yield a higher order compact interpolation. And in [14] a positivity-preserving limiter is introduced to enhance the robustness of the developed hybrid schemes. However there have been few efforts in the literature in constructing hybrid schemes that combine the MUSCL (upwind) method and the artificial viscosity (central) method. And therefore this work has developed the schemes that hybridize these two widely-used but very different FV methods. Note that this combination is easily ignored because either method of MUSCL and artificial viscosity, despite their inherent drawbacks, is equipped with shock-capturing capability. In the present work, we first enhance the accuracy of the existing methods by suggesting a simple and efficient improvement to the central schemes. Then the hybrid schemes are constructed by taking the advantages of the improved central schemes and a well-designed upwind scheme. At last our hybridization methodology [15] is built on a new concept of MOOD approach [16–18], which is radically different from previous work [8–10] since no indicator is applied. We remark that the resulting schemes are as compact as the underlying finite volume methods and by the current hybridization the defects from original central or upwind schemes are completely eliminated.

The rest of the paper is organized as follows. Section 2 briefly introduces the governing equation and the finite volume formulation applied in the work. Section 3 describes the details of the hybrid central-upwind schemes. Numerical tests are carried out in Section 4 and some conclusions are drawn in Section 5.

2. The governing equation and the finite volume discretization

2.1. Governing equations

By using the free stream density, sound speed, temperature and molecular viscosity, the non-dimensional NS equations can be written as

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \nabla \bullet \boldsymbol{F}_c(\boldsymbol{Q}) = \nabla \bullet \boldsymbol{F}_v(\boldsymbol{Q}, \nabla \boldsymbol{Q}) \tag{1}$$

where $\boldsymbol{Q} = (\rho, \rho u, \rho v, \rho w, \rho e)$ which are non-dimensional conservative variables, ρ is the density, u, v and w are the velocity components, e is the total energy per unit mass, $\boldsymbol{F}_c$ and $\boldsymbol{F}_v$ are the non-dimensional inviscid and viscous flux tensors, and t means non-dimensional time. The formula for the viscous flux is

$$\boldsymbol{F}_v = \begin{Bmatrix} 0 \\ \tau_{xx}\boldsymbol{i} + \tau_{xy}\boldsymbol{j} + \tau_{xz}\boldsymbol{k} \\ \tau_{yx}\boldsymbol{i} + \tau_{yy}\boldsymbol{j} + \tau_{yz}\boldsymbol{k} \\ \tau_{zx}\boldsymbol{i} + \tau_{zy}\boldsymbol{j} + \tau_{zz}\boldsymbol{k} \\ \Pi_x\boldsymbol{i} + \Pi_y\boldsymbol{j} + \Pi_z\boldsymbol{k} \end{Bmatrix}, \qquad \begin{aligned} \Pi_x &= u\tau_{xx} + v\tau_{xy} + w\tau_{xz} - q_x \\ \Pi_y &= u\tau_{xy} + v\tau_{yy} + w\tau_{yz} - q_y, \\ \Pi_z &= u\tau_{xz} + v\tau_{yz} + w\tau_{zz} - q_z \end{aligned}$$

$$\tau_{x_i x_j} = \left(\frac{M_\infty}{Re_\infty}\right) \mu \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]$$

$$q_{x_i} = -\frac{\boldsymbol{M}_\infty \mu}{Re_\infty(\gamma - 1)Pr}\frac{\partial T}{\partial x_i}$$

where $\tau_{x_i x_j}$ and q_{x_i} are the components of viscous stress tensor and heat flux, M_∞ and Re_∞ are free-stream Mach number and Reynolds number, μ is molecular viscosity coefficient, γ is the ratio of the specific heat, Pr is the Prandtl number and T is the temperature.

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