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Coupling finite element method with meshless finite difference method in thermomechanical problems

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ABSTRACT

This paper focuses on coupling two different computational approaches, namely finite element method (FEM) and meshless finite difference method (MFDM), in one domain. The coupled approach is applied in solving thermomechanical initial–boundary value problem where the heat transport in the domain is non-stationary. In this method, the domain is divided into two subdomains for FEM and MFDM, respectively. Contrary to other coupling techniques, the approach presented in this paper is defined in terms of mathematical problem formulation rather than at the approximation level. In the weak form of thermomechanical initial–boundary value problem (variational principle), the appropriate additional coupling integrals are defined a-priori. Subsequently, the FEM and the MFDM approximations, which may differ from each other, are provided to the formulation. It is assumed that there exists a very thin layer of material between the subdomains, which is not spatially discretized. The width of this layer may be considered the coupling parameter and it is the same for both, thermal and mechanical parts. Similar approach is applied to essential boundary conditions (e.g. prescribed temperature and displacements). Consequently, the consistent formulation of the mixed problem for the coupled FEM–MFDM method is derived. The analysis is illustrated with two- and three-dimensional examples of mechanical and thermomechanical problems.

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1. Introduction

This paper describes two-dimensional (2D) and three-dimensional (3D) non-stationary thermomechanical boundary problems. They are analysed by the original computational approach that couples the finite element method (FEM) and the meshless finite difference method (MFDM, [1–4]). The MFDM is a representative approach among a wide class of meshless methods [2,5–14]. FEM uses structural grids of elements (meshes) and unknown function approximation by means of the appropriate polynomial shape functions, built upon finite elements and associated with nodal degrees of freedom. On the other hand, MFDM, as one of the oldest meshless methods, may use both regular meshes or totally arbitrarily irregular clouds of nodes, without any imposed structure, like finite element, regular mesh or mapping restrictions. Moreover, function approximation is prescribed by nodes only, replacing the differential operators with difference ones. Various types of local function approximation may be distinguished, being the fundamental criterion for meshless methods classification. While the MFDM is based upon the Moving Weighted Least Squares [2,15,16], other commonly applied techniques include reproducing kernel [17–19], radial basis functions and partition of unity approaches [20–25].

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The motivation of tackling this particular scientific subject may be supported by the existence of such problems in which it is more convenient and effective to incorporate two different methods in different parts of the domain. The sort of problems where the coupling is justified are for example the problems in which selected parts of the domain are under rapid change of subjected load, boundary conditions and/or material parameters (e.g. due to temperature dependency). In such problems, applying a meshless approach (especially the MFDM) to those regions seems to be reasonable due to its higher flexibility in nodes generation, super-convergence of solution derivatives, as well as element-free determination of the approximation base (in general). Moreover, non-linear problems, in which the basic linearized problem has to be solved many times, or situation in which the approximation mesh/cloud of nodes requires frequent refinements (e.g. due to the adaptation technique or shrinking of the material), are other examples.

The coupling of FEM with other methods for computational analysis of boundary value problems (especially with meshless methods) is not new as it reaches early seventies of the previous century [26–28]. Since then, this issue has been later investigated by many other researchers (e.g. [29,30]) and it is still under development nowadays [31–35]. In most cases, the main idea behind the coupling methods is either to eliminate or reduce drawbacks of one method (e.g. time-consuming mesh generation, low rate of derivatives convergence) or to use the advantages of other method (super-convergence, least squares smoothing, independent integration mesh and so on) in more effective manner. Those all investigations resulted in various coupling methods which can be applied at various levels of analysis, i.e.:

- different methods in disjoint subdomains with appropriate transition zones [29,30,36],
- domain discretization by FEM and generation of approximation schemes by MFDM (e.g. numerical differentiation, [26–28]),
- general postprocessing of FEM results by means of meshless approximation (e.g. application of the Moving Weighted Least Squares, for a-posteriori error analysis, [2,15,16,37]).

The main aim of this paper is to present the effective coupling method for 2D and 3D cases applied to thermomechanical problems with non-stationary heat transport. The thermomechanical or fluid–solid structure coupled problems involving meshless approaches on different levels of numerical analysis have been investigated by many researchers, e.g. [38–40]. The FEM–MFDM coupled method which is analysed in this work, is based on the approach proposed in [41] and selected thermomechanical relations as well as coupled variational formulation are taken from the paper [42]. It is assumed that the domain under consideration is divided into two or more subdomains to which FEM or MFDM (or any other meshless method) are applied separately. Each subdomain may be multi-compact, i.e. may consist of several disjoint parts. The subdomains may have different mesh/nodes density, degrees of freedom as well as approximation schemes. Neither additional transition zones in the mesh of nodes/elements nor specially dedicated shape functions are required.

For the sake of clarity, the domain is divided into two subdomains: for FEM and MFDM, respectively. The border between those subdomains constitutes a surface in 3D or a curve in 2D. It is presumed that the width of the border has a non-zero value. On the other hand, this width is very small when compared to other dimensions, therefore it can be neglected in the spatial discretization. Within this thin border zone, the consistent physically based thermo-mechanical relations are applied guaranteeing that both the problem formulation and the approximate solution are continuous in the whole domain. It should be emphasized at this point, that although the coupling approach is general, the coupling relations have to be adjusted to the considered problem. The coupling is provided at the level of variational formulation in contrary to other coupling approaches where special approximations are prepared, e.g. [26–30,33,34,36,37,43,44]. In consequence, in this work, discretizations and approximations on FEM and MFDM subdomains are constructed independently and even then the obtained coupled approximate solution remains continuous. The proposed approach may be considered as one of the domain decomposition methods [45,46], convenient in parallel computing. However, neither the application of time-consuming iterations between disjoint subdomains, nor use of Lagrange multipliers (additional degrees of freedom) is required to preserve the accuracy of the solution.

In the authors' previous paper [41], only the scalar boundary problem (stationary heat flow) has been considered. It was clearly indicated that the coupling method is appropriate and it may be successfully applied to the analysis of 2D and 3D problems or even higher dimension cases. The same coupling method is used here for more complex 2D and 3D vector boundary problems i.e. linear elasticity as well as stationary and non-stationary thermoelasticity problems. However, it results in more complex relations on the subdomains' border and both unknown fields (temperature, displacements) are discretized independently in each subdomain. The application of the coupling method to the non-stationary coupled thermo-mechanical problem is the main original element of this work.

The appropriate technique for applying Dirichlet boundary condition to heat conduction problem was presented in [41]. The same technique is adapted in this paper and subsequently extended to the elasticity and thermoelasticity problems. It allows for fulfilment of the Dirichlet boundary conditions for FEM and MFDM subdomains in the same manner. This approach seems to be especially convenient in the case of non-singular weight functions in meshless approximation (lack of a-priori interpolation properties). Therefore, it enables effective imposition of the Dirichlet conditions to the outer boundary for both interpolating (FEM and MFDM) and non-interpolating approximations (other meshless methods). It may be noted that approach presented in [41], has been extended to bounded discontinuous approximations in [47,48].

Two non-conforming approaches (i.e. the non-conformity is in the discretizations and approximations) are coupled together in this paper. However, the coupling method differs from other commonly applied coupling techniques for non-conforming problems, like mortar, penalty and Nitsche methods. In the mortar methods, additional unknowns are used

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