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Oscillation criteria for impulsive partial fractional differential equations

A. Raheem *, Md. Maqbul

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India
Department of Mathematics, National Institute of Technology Silchar, Assam - 788010, India

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ABSTRACT

In this paper, we established some sufficient conditions for oscillation of solutions of a class of impulsive partial fractional differential equations with forcing term subject to Robin and Dirichlet boundary conditions by using differential inequality method. As an application, we included an example to illustrate the main result.

1. Introduction

The fractional differential equations are used to describe mathematical models of numerous real processes and phenomena studied in many areas of science and engineering such as population dynamics, neural networks, industrial robotics, viscoelasticity, electric circuits, optimal control, biotechnology, economics. It should be noted that most of papers and books on fractional differential equations and impulsive partial differential equations are devoted to the existence and uniqueness of solutions. There are only few papers dealt with the oscillatory behavior of solutions. In recent years, the oscillatory behavior of various classes of fractional differential equations and impulsive partial differential equations have been investigated by many authors [1–10].

In [1,6,10], the authors have obtained some sufficient conditions for the oscillatory behavior of fractional differential equations with damping terms. In [2], the authors have investigated the oscillatory behavior of solutions of a nonlinear fractional partial differential equation with damping and forced term subject to Robin boundary conditions. Some new developments in the oscillatory behavior of solutions of fractional ordinary differential equations have been investigated by the authors in [5,7].

Recently, some authors have studied the oscillatory behavior of solutions of partial fractional differential equations and impulsive partial differential equations [8–16]. In [9], Li has established some sufficient conditions for the forced oscillation of certain partial fractional differential equations by using the techniques of the differential inequalities. In [11], Prakash and Harikrishnan have established several sufficient conditions for H-oscillation of solutions of impulsive vector hyperbolic differential equations with delays. For more developments on H-oscillation of solutions impulsive parabolic and hyperbolic partial differential equations, we refer the reader to [12] and the references cited therein.

* Corresponding author at: Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.
E-mail addresses: araheem.iitk3239@gmail.com (A. Raheem), maqboolkareem@gmail.com (Md. Maqbul).

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In [13], Fu and Zhang have investigated several sufficient conditions for the following class of nonlinear impulsive delay hyperbolic systems with Robin and Dirichlet boundary conditions:

$$\begin{cases} u_{tt} = a(t)\Delta u(t,x) + b(t)\Delta u(t-\sigma,x) - p(t,x)u(t,x) - q(t,x)f[u(t-r,x)] + g(t,x), & t \neq t_k, \\ u(t_k^+,x) - u(t_k^-,x) = Iu(t_k,x,u), & k = 1,2,3\ldots, \\ u_t(t_k^+,x) - u_t(t_k^-,x) = Ju(t_k,x,u), & k = 1,2,3\ldots, \end{cases}$$

where Δ is the Laplacian in R^n, Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$, $a,b \in PC[R_+,R_+]$, $p,q \in PC[R_+\times\overline{\Omega},R_+]$, PC denote the class of functions which are piecewise continuous in t with discontinuities of first kind only at $t=t_k$, $f \in C[R,R]$, the forcing term $g \in PC[R_+\times\overline{\Omega},R]$, σ, r are positive constants and $I,J : R_+\times\overline{\Omega}\times R \to R$.

For earlier works on oscillatory behavior of solutions of impulsive hyperbolic partial differential equations, we refer the readers to [14,15] and the references cited therein.

Motivated by all the above works, in this paper, we consider the oscillatory behavior of solutions of the following fractional impulsive differential equation:

$$\begin{cases} D_{+,t}^{\beta}u(x,t) + a(t)D_{+,t}^{\beta-1}u(x,t) = b(t)\Delta u(x,t) + \sum_{k=1}^{m} c_k(t)\Delta u(x,t-\tau_k) - F(x,t), & t \neq t_j \\ D_{+,t}^{\beta-1}u(x,t_j^+) - D_{+,t}^{\beta-1}u(x,t_j^-) = \sigma(x,t_j)D_{+,t}^{\beta-1}u(x,t_j), & j = 1,2,3,\ldots, (x,t) \in \Omega\times R_+ = G, \end{cases} \tag{1}$$

where $a,b,c_k \in PC[R_+,R_+]$ and forcing term $F \in PC[\overline{\Omega}\times R_+,R_+]$, where PC denotes the class of functions which are piecewise continuous functions in t with discontinuities of first kind only at $t=t_j, j=1,2,\ldots$ and left continuous at $t=t_j$, $\beta \in (1,2)$ is a constant, Δ is the Laplacian in R^n, Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$ and $\overline{\Omega} = \Omega\cup\partial\Omega$.

We shall consider two kinds of boundary conditions

$$\frac{\partial u(x,t)}{\partial N} + f(x,t)u(x,t) = 0, \quad (x,t) \in \partial\Omega\times R_+,\ t \neq t_j \tag{2}$$

and

$$u(x,t) = 0, \quad (x,t) \in \partial\Omega\times R_+,\ t \neq t_j, \tag{3}$$

where $f \in PC[\partial\Omega, R^+]$, N is the unit out normal vector to $\partial\Omega$.

The organization of the rest of this paper is as follows. In Section 2, we briefly state some basic definitions and assumptions which will be used in Section 3. In Section 3, we obtain several oscillation criteria for impulsive fractional differential equations with two type of boundary conditions, Robin and Dirichlet boundary conditions, by using differential inequality method.

2. Assumptions and preliminaries

Throughout this article, we assume that the following conditions hold:

(H1) $\sigma : \overline{\Omega}\times R_+ \to R_+$ such that

$$\sigma(x,t_j) \le \alpha_j.$$

(H2) The given numbers

$$0 < t_1 < \cdots < t_k < \cdots$$

are such that

$$\lim_{k\to+\infty} t_k = +\infty.$$

(H3) The solution $u(x,t)$ of the problem (1) with (2) or (3) and $D_{+,t}^{\beta-1}u(x,t)$ are piecewise continuous with discontinuities of first kind only at $t=t_j$, and left continuous at $t=t_j$, $j=1,2,3,\ldots$, i.e., at the moments of impulse the following relation is satisfied

$$D_{+,t}^{\beta-1}u(x,t_j^-) = D_{+,t}^{\beta-1}u(x,t_j), \quad j=1,2,3,\ldots.$$

Definition 2.1. A nonzero solution $u(x,t)$ of the problem (1), (2) or (1), (3) is said to be nonoscillatory in the domain G if there exists a number $\tau \ge 0$ such that $u(x,t)$ has a constant sign for $(x,t) \in \Omega\times[\tau,+\infty)$. Otherwise, it is said to be oscillatory.

Definition 2.2 ([18]). The Riemann–Liouville fractional derivative of order $\alpha > 0$, for a function $f(t)$ on the half axis R_+ can be defined as

$$D_{+,t}^{\alpha}f(t) = \frac{1}{\Gamma(\lceil\alpha\rceil-\alpha)}\frac{d^{\lceil\alpha\rceil}}{dt^{\lceil\alpha\rceil}}\int_0^t (t-\tau)^{\lceil\alpha\rceil-\alpha-1}f(\tau)d\tau$$

provided the right hand side is pointwise defined on R_+, where $\lceil\alpha\rceil$ is the ceiling function of α function.