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Alternating direction method of multiplier for a unilateral contact problem in electro-elastostatics[☆]

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ABSTRACT

We study an alternating direction method of multiplier (ADMM) applied to a unilateral frictional contact problem between an electro-elastic material and an electrically non conductive foundation. The frictional contact is modeled by the Tresca friction law. The resulting coupled problem is non symmetric and non coercive. By eliminating the electric potential, we obtain a symmetric and coercive problem which can be reformulated as a convex minimization problem. We then apply an alternating direction method of multiplier for the numerical approximation. To avoid explicit matrices inverse (due to the elimination of the electric potential) we use a preconditioned conjugate gradient algorithm as an inner solver. Numerical experiments are proposed to illustrate the efficiency of the proposed approach.

1. Introduction

During the last decades an important interest is devoted to the contact problem involving piezoelectric materials. General models for elastic materials with piezoelectric effects can be found in [1–3]. The frictional and frictionless contact problems for electro-elastic materials are studied in [4–7]. Analysis and numerical simulation of contact with or without friction for piezoelectric materials can be found in [8,9] and references therein. The coupling between the mechanical and electrical properties leads to a non symmetric and non coercive problem. The numerical analysis of this problem is not easy and in most cases a finite element method is used [10,4,11]. More recently, Essoufi et al. [9], propose a numerical approximation of a unilateral contact problem in electro-elastostatics with Tresca friction and conductive foundation. The splitting between the mechanical and electrical subproblems is obtained by a successive iterative method. An alternating direction method of multiplier is then applied to the mechanical subproblem as an inner solver. The convergence of the successive iterative method is proved under some assumptions.

We propose in this paper a new approach for the numerical approximation of a unilateral contact problem with Tresca friction and non conductive foundation. We take advantage of the linear coupling between mechanical and electrical effects to eliminate the electrical potential. We get a new symmetric and coercive problem similar to a contact problem in elasticity which can be reformulated as a convex minimization problem. We then apply alternating direction method of multiplier (ADMM) for the numerical approximation. To avoid explicit computation of matrices inverse (due to the elimination of the electrical potential), we use a suitable preconditioned conjugate gradient algorithm.

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The paper is organized as follows. In Section 2, we state the problem, we give the weak formulation and we use our approach to decouple the problem, the minimization problem is posed. In Section 3, we establish the continuous and algebraic augmented Lagrangian formulation and we state the saddle point problem. In Section 4, we set the alternating direction method of multiplier algorithm. In Section 5, some numerical experiments are carried out to illustrate the behavior of the proposed algorithm.

2. Model problem

In this section we state the model problem for a unilateral contact problem with the Tresca friction law between a piezoelectric material and an electrically non conductive rigid foundation.

2.1. Mechanical problem

We consider a piezoelectric body occupying in its initial (undeformed) configuration a bounded domain $\Omega \subset \mathbb{R}^2$ with a smooth boundary $\partial\Omega = \Gamma$. We assume that Γ is divided into three disjoint parts, i.e. $\Gamma = \Gamma_D \cup \Gamma_N \cup \Gamma_C$. We assume that, on Γ_D, the body is clamped and the electric potential vanishes. Surfaces traction and surface electric charge act on Γ_N. On Γ_C the body may come in frictional contact with a rigid foundation which is electrically non conductive. We assume linear piezoelectricity, then the strain tensor ε and the electric intensity vector E are given by

$$\varepsilon(u) = \frac{1}{2}(\nabla u + (\nabla u)^\top) \quad \text{and} \quad E(\varphi) = -\nabla\varphi,$$

where $u = (u_1, u_2)$ is the displacement field and φ the electric potential. The constitutive relations for the piezoelectric material, coupling the stress tensor σ and the electric charge density displacement D, are

$$\sigma = \mathcal{A}\varepsilon(u) - \mathcal{E}^* E(\varphi) \quad \text{in } \Omega,$$
$$D = \mathcal{E}\varepsilon(u) + \beta E(\varphi) \quad \text{in } \Omega,$$

where $\mathcal{A} = (a_{ijkl})$ is the (fourth-order) symmetric, bounded (i.e. $a_{ijkl} \in L^\infty(\Omega)$) and coercive elasticity tensor, $\mathcal{E} = (e_{ijk})$ is the (third-order) bounded (i.e. $e_{ijk} \in L^\infty(\Omega)$) piezoelectric tensor with $e_{ijk} = e_{ikj}$, and $\beta = (\beta_{ij})$ is the symmetric and coercive electric permittivity. We use $\mathcal{E}^*$ to denote the transpose of the tensor $\mathcal{E}$ defined by

$$\mathcal{E}\sigma \cdot v = \sigma : \mathcal{E}^* v, \quad \forall \sigma \in \mathbb{S}^2,\ \forall v \in \mathbb{R}^2.$$

Let ν be the unit outward normal to Ω on Γ. The displacement field and the stress tensor can be decomposed in normal and tangential components as follows

$$u_\nu = u \cdot \nu, \qquad u_\tau = u - u_\nu \nu,$$
$$\sigma_\nu = (\sigma\nu) \cdot \nu, \qquad \sigma_\tau = \sigma\nu - \sigma_\nu \nu.$$

The mechanical and electrical equilibrium equations are given by

$$-\operatorname{div}(\sigma) = f \quad \text{in } \Omega, \tag{2.1}$$
$$\operatorname{div}(D) = q \quad \text{in } \Omega, \tag{2.2}$$
$$\sigma\nu = f_s, \qquad D\nu = q_s \quad \text{on } \Gamma_N, \tag{2.3}$$
$$u = 0, \qquad \varphi = 0, \quad \text{on } \Gamma_D, \tag{2.4}$$

where $f \in L^2(\Omega)^2$, $q \in L^2(\Omega)$, $f_s \in L^2(\Gamma_N)^2$ and $q_s \in L^2(\Gamma_N)$.

On Γ_C the body may come in frictional contact with a rigid foundation which is electrically non conductive. Let $g \in L^\infty(\Gamma_C)$ be the normalized gap between Ω and the rigid foundation. The unilateral contact conditions are then

$$u_\nu - g \le 0, \qquad \sigma_\nu \le 0 \quad \text{and} \quad (u_\nu - g)\sigma_\nu = 0 \quad \text{on } \Gamma_C, \tag{2.5}$$

and the friction conditions are (Tresca friction)

$$\begin{cases} |\sigma_\tau| \le s, \qquad |\sigma_\tau| < s \implies u_\tau = 0 \quad \text{on } \Gamma_C \\ \qquad\qquad |\sigma_\tau| = s \implies \exists\lambda \ge 0 \quad \text{such that } u_\tau = -\lambda\sigma_\tau \text{ on } \Gamma_C, \end{cases} \tag{2.6}$$

where $s \in L^\infty(\Gamma_C)$ is the slip bound. In this paper, we also consider the Coulomb friction conditions

$$\begin{cases} |\sigma_\tau| \le \nu_f|\sigma_\nu|, \qquad |\sigma_\tau| < \nu_f|\sigma_\nu| \implies u_\tau = 0 \quad \text{on } \Gamma_C \\ \qquad\qquad |\sigma_\tau| = \nu_f|\sigma_\nu| \implies \exists\lambda \ge 0 \quad \text{such that } u_\tau = -\lambda\sigma_\tau \text{ on } \Gamma_C. \end{cases} \tag{2.7}$$

Conditions (2.7) will be approximated using a sequence of Tresca friction problems. The electric contact condition is

$$D \cdot \nu = 0 \quad \text{on } \Gamma_C, \tag{2.8}$$

which means that the foundation is not electrically conductive.