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An *h*-adaptive mortar finite element method for finite deformation contact with higher order *p* extension

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ABSTRACT

In this work we present a generalization of the mortar segment-to-segment method for finite deformations contact to an *h*-adaptive version with possible *p* extension, i.e. using higher order approximation. We recall the main ideas of the mortar algorithm and present the key aspects of adaptivity: error estimation and an *h*-adaptive strategy. The *p* extension exploits the feature of the *hp*-adaptive code in which the contact solver is implemented to handle meshes with nodes of nonuniform orders. We use it to set interior nodes to higher order while leaving linear boundary contact nodes which can be processed by the standard mortar algorithm. Accuracy of elements with low order nodes is restored by adequate subdividing of these elements. Adaptivity and *p* extension are illustrated with a few numerical tests.

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1. Introduction

Numerical simulation of contact problems is a challenging area of computational mechanics. This specially concerns large deformation multibody contact. Numerous versions of contact algorithms have been presented, for instance, in the monographs of Wriggers [1] and Laursen [2]. Unlike in many other fields of PDE simulations not much investigation was devoted to the aspect of error control and adaptivity of discretizations of contact problems. The aim of this work is to apply error estimation and *h*-adaptivity for large deformation contact problems and to indicate the possibility of using higher order analysis, exceeding the basic expectation of quadratic approximation, and letting envision full *hp*-adaptivity.

Our investigations are focused on generalization of the mortar segment-to-segment (STS) contact method proposed by Puso and Laursen [3] and extended by Popp et al. [4], to an *h*-adaptive method. We develop an *h*-adaptive strategy based on *a posteriori* error estimation technique and the methodology of performing computations on the irregular meshes (elements with hanging nodes) which allow one for a full flexibility of adapting the meshes to possibly irregular solutions.

In mortar STS approach the contact conditions are enforced in integral, variational manner as opposed to the most popular (especially in commercial applications) alternative node-to-segment (NTS) method which uses rather the collocation approach enforcing contact conditions between selected points, usually nodes of elements of one of contacting surface and elements (segments) of the other contacting boundary.

The reason of investigating in context of adaptivity the mortar STS rather than NTS approach is not accidental. The major aspect of using adaptive methods is the possibility of generating strongly graded meshes leading to meshes with elements of essentially different sizes. If it happens that using the NTS method one enforces the contact conditions at nodes of some

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large element and an opposite contacting mesh of very fine elements then the phenomenon similar to enforcing pointwise Dirichlet boundary conditions occurs: large deformation and accompanying stresses arise which are difficult to control in a nonlinear solution scheme. Secondly, pointwise satisfaction of contact conditions is not easy to interpret in designing *a posteriori* error estimation methods.

On the opposite side, the mortar STS method is free of these shortcomings. The density of integration points which are involved in satisfaction of contact is always dictated by the smaller elements, while the forces of interaction of the bodies are considered as distributed quantities (tractions) which can be easily incorporated for error estimation algorithms.

We implemented the contact algorithm within the general purpose *hp*-adaptive code *hp3d* developed by Demkowicz et al. [5]. It allows us for combined *h* and *p* refinements, the former consisting in local subdividing the elements while the latter in locally enriching their approximation orders, in addition, both modifications can be anisotropic. The flexibility of the kernel program allows us to achieve an additional advantage, especially appreciated in the context of contact methods: to use the higher order approximation. Thus far schemes with quadratic elements were designed by Puso et al. [6] and by Popp et al. [7]. These methods use a mechanism of linking the quadratic approximation on contact surfaces to linear one on twice smaller elements which in turn are involved in geometrical analysis of contact as for linear approximation. Applying the *hp*-code allows us to use even higher than quadratic approximation: it consists just in typical and standard operation in *hp3d* of enriching the interior nodes to a selected higher order and leaving nodes on contact surfaces of linear order. The deteriorated (relative to the interior) quality of approximation on the boundary can be easily improved just by subdividing the elements of reduced order (a required number of times).

We illustrate the adaptive method with a number of numerical simulations involving nearly incompressible (rubber-like) materials, for which we apply a mixed two field, displacement–pressure, approach. In one elementary test we demonstrate approximation with elements of order $p = 4$, in the remaining examples we use linear *h*-adaptive meshes and piecewise uniform meshes of order $p = 2$ elements. One of the experiments demonstrates possibility of applying our technique to numerical simulation of the medical procedure of balloon angioplasty with stenting where also elasto-plastic response of one of the components (the stent) is modeled.

In subsequent sections we discuss formulation of the contact problem for finite elasticity, the finite element discretization and the details of the mortar technique including consistent linearization. Next we discuss the issues of adaptivity: the error estimation and aspects of the *h*-adaptive strategy. Finally, we illustrate the designed techniques with numerical examples.

2. Formulation

2.1. Finite strain elasticity and plasticity

We use standard tools of material description of finite deformation of a solid body. We denote by $\boldsymbol{X}$ the location of a material point in reference configuration while by $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X}, t)$ its location after deformation at time instant t, and $\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{X}$ is the displacement. We define the gradient of deformation and the right Cauchy–Green deformation tensors:

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}, \qquad \boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{F}. \tag{1}$$

The internal forces are expressed by the second Piola–Kirchhoff stress tensor $\boldsymbol{S}$ which for a hyperelastic material can be derived from the strain energy function:

$$\boldsymbol{S} = 2\frac{\partial \Psi(\boldsymbol{C})}{\partial \boldsymbol{C}}. \tag{2}$$

We consider nearly incompressible materials with possible anisotropy resulting, for instance, from presence of one or two families of fibers for which the following sum of energy contributions is often postulated [8]:

$$\Psi = U(J) + \bar{\Psi}_{iso}(\bar{\boldsymbol{C}}) + \bar{\Psi}_{aniso}(\bar{\boldsymbol{C}}, \boldsymbol{A}_i), \quad i = 1, 2, \tag{3}$$

where $J = \det(\boldsymbol{F})$, $\bar{\boldsymbol{C}} = J^{-2/3}\boldsymbol{C}$, and $\boldsymbol{A}_i := \boldsymbol{M}_i \otimes \boldsymbol{M}_i$ are structural tensors defined by up to two families of material directions of fibers $\boldsymbol{M}_i(\boldsymbol{X})$, $|\boldsymbol{M}_i| = 1$. Particular forms of contributions in (3) corresponding to the volumetric and isochoric deformations of isotropic and anisotropic materials are discussed later. We also consider elasto-plastic behavior of a solid body in the finite strain regime, following Simo and Hughes [9]. In this case we limit ourselves to materials with isotropic elastic responses corresponding to the strain energy:

$$\Psi = U(J) + \frac{\mu}{2}(\bar{I}_1 - 3), \tag{4}$$

where material parameter $\mu > 0$ and $\bar{I}_1 = \mathrm{tr}[\bar{\boldsymbol{C}}]$. For such materials we consider plasticity model with a von Mises–Hill yield function with isotropic linear hardening. In this model multiplicative decomposition into plastic and elastic parts of the deformation gradient $\boldsymbol{F}$ is considered [9]:

$$\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p, \tag{5}$$

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