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A fast numerical method to price American options under the Bates model

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ABSTRACT

We consider the problem of pricing American options in the framework of a well-known stochastic volatility model with jumps, the Bates model. According to this model, the price of an American option can be obtained as the solution of a linear complementarity problem governed by a partial integro-differential equation. In this paper, a numerical method for solving such a problem is proposed. In particular, first of all, using a Bermudan approximation and a Richardson extrapolation technique, the linear complementarity problem is reduced to a set of standard linear partial differential problems (see, for example, Ballestra and Sgarra, 2010; Chang et al. 2007, 2012). Then, these problems are solved using an ad hoc pseudospectral method which efficiently combines the Chebyshev polynomial approximation, an implicit/explicit time stepping and an operator splitting technique. Numerical experiments are presented showing that the novel algorithm is very accurate and fast and significantly outperforms other methods that have recently been proposed for pricing American options under the Bates model.

1. Introduction

One of the most popular models for pricing financial options is the so-called Black–Scholes model, according to which the price of the underlying asset is described by a geometric Brownian motion with constant volatility. However, many empirical studies have revealed that the volatility is far from being constant (see, or example, [1] and references therein), and that the asset prices are often subject to jumps, i.e. abrupt and large time variations (see [2,3]). A noticeable generalization of the Black–Scholes model which allows one to incorporate both non-constant (stochastic) volatility and jumps in the asset price has been developed by Bates in [4].

In this paper, we consider the problem of pricing American options on an underlying described by the Bates model. Unlike European options, which can be exercised at the expiration date only, American options give to the holder the right to buy or sell the underlying instrument at any time prior to maturity. Note that it is rather interesting to consider American options because the majority of options that are traded on the financial markets are of this type.

From the mathematical standpoint, the price of an American option under the Bates model can be obtained solving a linear complementarity partial integro-differential problem in two spatial variables, namely the price and the variance of the underlying asset. Such a problem does not have an exact closed-form solution and thus requires numerical approximation. To this aim, some authors have proposed either finite difference or finite element methods. In particular, Toivanen [5] has

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employed a finite difference scheme combined with a componentwise splitting approach and the projected successive over relaxation (PSOR) technique, Chiarella et al. [6] have proposed a finite difference scheme coupled with the method of lines and a PSOR procedure, Salmi et al. [7] have developed a finite difference scheme based on the projected algebraic multigrid method presented in [8], Salmi et al. [9] have employed a finite difference scheme coupled with either the projected algebraic multigrid method as in [8] or a linear complementarity approach based on LU decomposition and operator splitting, Ballestra and Sgarra [10] have used a finite element method combined with an operator splitting approach and a Richardson extrapolation procedure, Broadie and Kaya [11] and D'Ippoliti et al. [12] have resorted to Monte Carlo simulation techniques.

In the present manuscript, we depart from the existing finite difference and finite element literature, and propose a pseudospectral method based on Chebyshev polynomial approximation. The Chebyshev polynomial approximation is used in various areas of science and engineering for solving a number of partial differential equations (see, e.g., [13–18]) or other kinds of problems (see, e.g., [17,19,20]), and it has also been applied in mathematical finance. In particular, Chebyshev pseudospectral methods have been proposed by [21] in order to price Asian options under the standard Black–Scholes model, by [22–24] in order to price European options under a jump–diffusion model (without stochastic volatility) and by [25] in order to price European options under a Black–Scholes model with time-varying interest rates.

The Chebyshev pseudospectral approach has the advantage of being computationally very efficient, as it yields high levels of accuracy even if it is employed in conjunction with relatively coarse grids (see, e.g., [26,27,14,28]). Nevertheless, it is a global approximation method (the stencil to be used in order to compute derivatives at a given point extends over all the mesh points), which thus has the disadvantage of requiring the inversion of large system matrices. To remedy this inconvenient, in the present paper we employ the technique proposed in [29], that is by applying a suitable implicit–explicit operator splitting scheme, we transform the original partial differential problem to a set of small linear systems, such that the inversion of large system matrices is avoided.

In addition, in order to properly take into account the possibility of early exercise typical of the American options, the Chebyshev approximation is combined with an ad-hoc extrapolation method. Precisely, following for example [10,30], first of all, the American option is approximated by a set of Bermudan options, i.e. options that can be exercised only at a discrete set of dates. Then, an accurate estimation of the American option price is obtained by (repeated) Richardson extrapolation of the prices of the aforementioned Bermudan options (as the American option price can actually be thought of as the limit of a sequence of Bermudan options as the number of exercise dates approaches infinity). This is a very efficient approach for taking into account the possibility of early exercise. In fact, in contrast to other methods applicable to models with two stochastic factors, such as linear complementarity methods (see for example [31,32]), or the penalty method (see for example [33,34,31,35–38]), the Richardson extrapolation technique does not require one to perform fixed-point iteration procedures.

Numerical experiments are presented showing that the discretization method resulting from the combination of the aforementioned techniques (the operator splitting procedure, the Chebyshev pseudospectral approximation, the Richardson extrapolation) achieves high computational performances. In particular, the novel algorithm outperforms an approach presented in [39] and behaves fairly better than the method proposed by [10], which is currently one of the most efficient numerical schemes for pricing American options under the Bates model (at least to the best of our knowledge).

Finally, it is worth recalling that in the literature options of complex type (such as American or barrier options) are often priced using numerical approaches based on integral equations or convolution techniques, see, e.g., [40–47]. These schemes can yield extremely accurate results, but their extension to pricing American options under stochastic volatility is not straightforward. In fact, one would have to compute integrals defined on a set of the (S, v) plane whose boundary (the so-called early exercise boundary, see Section 2 as well as [48–50]) is a curved line. In addition, the aforementioned integral approaches are not very flexible, as they can be used only in the case where the transition probability density function of the underlying option price (or at least its characteristic function) is explicitly known. On the contrary, the pseudospectral method proposed in the present paper is fairly general and as such could be applied to a variety of models also different from the standard Bates model (just to make some example, we could think to add a CEV dynamics [51] in the stochastic differential equation, or to consider jumps that are not log-normally distributed, see, for instance, [52]).

The paper is structured as follows: in Section 2 the partial differential problem that yields the price of an American option under the Bates model is shown; in Section 3 the numerical method is developed; in Section 4 the main numerical results are presented and discussed; finally, in Section 5 some conclusions are drawn.

2. The mathematical problem

According to the Bates model, the price and the variance of a risky asset, denoted S and v, respectively, satisfy the following stochastic differential equations:

$$dS(t) = \mu S(t) + \sqrt{v(t)}S(t)dW_1(t) + (\eta - 1)S(t)dJ(t), \quad (1)$$

$$dv(t) = \gamma(\theta - v(t)) + \sigma\sqrt{v(t)}dW_2(t), \quad (2)$$

where μ, γ, θ, σ are constant parameters and W_1 and W_2 are standard Wiener processes. In particular, the correlation between W_1 and W_2 is assumed to be constant and is hereafter denoted by ρ. Moreover, J is a Poisson process (independent of W_1 and W_2) whose (constant) intensity is denoted by λ and η is a random variable measuring the jump amplitude. Following

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