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An advanced and adaptive mathematical function for an efficient anisotropic image filtering

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ABSTRACT

Image de-noising and enhancement are becoming increasingly widespread. Anisotropic diffusion is one of the greatest techniques that are used to remove noise of the image, while keeping the important information unvaried. Nevertheless, it suffers from many drawbacks such as the image blurring. This work is concerned with a new anisotropic diffusion approach founded on an innovative mathematical function for noise reducing and edge preserving. The entire contribution can be observed from the experimental results which have shown that the new proposed anisotropic scheme is not only able to remove the noise efficiently but also to preserve the content in the denoised image.

1. Introduction

Denoising aims to reduce the degree of sharp transitions in a distorted image. It tends to improve the image quality for better interpretation and data extraction. For this reason, de-noising image is becoming a fundamental operation in image processing. Many algorithms have been presented in the literature. For example, partial differential equations (PDEs) have shown significant efficiency in the field of image denoising. Based on these PDEs equations, many methods have extremely developed as image filtering techniques. These methods can perform well quite on image cleaning and edge preserving. However, they have some shortcoming. Koenderink [1] showed that the heat diffusion equation is an isotropic diffusion that causes the edge blur. In order to avoid this shortcoming of linear diffusion filtering, Perona and Malik [2] proposed a non-uniform smoothing technique which is called "Anisotropic diffusion 'AD'". It was the first formulation of the heat diffusion equation applied to digital image aiming to protect the image properties and preserving the edges by reducing the diffusivity near these locations. Following the original proposal by Perona and Malik, diverse approaches based anisotropic diffusion have been presented for edge detection [3], noise reducing [4]. Despite the AD has impressive results in image filtering, it suffers from many drawbacks like the artifact staircases [5]. To overcome this problem, many regularized approaches have been widely published. In [6,7], different Variational formulations expound the mathematical behavior of the AD. Furthermore, a lot of research works have been done in the image enhancement field [8,9]. Catté et al. [6] proposed a selective cleaning process wherein the gradient computation is based on a Gaussian kernel. Based on a directional diffusion that performs in one direction than another, Weickert [10] suggested the diffusion tensor. Many other techniques have been extensively studied as efficient approaches for image denoising such as the Total Variation denoising model (TV) [11] and its derived techniques [12,13]. Anisotropic diffusion filtering is an iterative process which depends on some parameters such as the conductance function as well as the time parameter. Hence, various works have been suggested in order to optimize

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these parameters for better de-noising results [3,14,15]. Image de-noising based on high order technique has been developed [16–18]. To deal with noise removal and edge preservation simultaneously, many other techniques via anisotropic diffusion based on the gradient vector flow [19,20] have been incorporated. Despite this variety of image denoising techniques based on anisotropic diffusion, many other approaches either anisotropic or not, have been formulated [21–25] which aim to well removing noise, preserving the image content and avoiding the staircasing effect.

We have compared our new algorithm to other recent methods such as Non Local-means algorithms [26–31] and BM3D based method [32].

The outline of this work is as follows: in Section 2, we will review the Anisotropic Diffusion concept in image restoration and denoising. The proposed model is described and analyzed in Section 3. Section 4 describes the implementation of the new mathematical function in the diffusion scheme. The contribution of the new methodology with the experimental results and the related discussions are exposed in Section 5. Section 6 concludes this work.

2. Anisotropic diffusion concept

2.1. Overview of Perona and Malik model

Image enhancement is an important operation in image processing. Linear diffusion based PDE equations are an efficient tool for image denoising. Thus, noise can be reduced by convolving the image by a Gaussian kernel. Assuming $I(x, y)$ is the intensity of a pixel $p(x, y)$ at the location (x, y) in an image I:

$$\frac{\partial I}{\partial t} = \begin{cases} div(c\nabla I(x,y,t)) = c \cdot div(\nabla I(x,y,t)) = c(I_{xx}(x,y,t) + I_{yy}(x,y,t)) \\ I(x,y,0) = I_0 \end{cases} \tag{1}$$

where c represent the scalar constant diffusivity I_0 is the initial distorted image, $I(x, y, t)$ is the image obtained after a diffusion time t. This shows that the diffusion is the same in all directions without taking into account the local properties of the image. This linear isotropic process is extensively used to remove noise, but it suffers from many disadvantages such as:

- Smoothing details: it not only smoothes the noise but also the edges, which makes the image blurred and difficult to identify and to analyze.
- Edge dislocation: it can dislocate edges when moving from finer to coarser scales.

In order to overcome these drawbacks, many techniques based on anisotropic diffusion have been suggested over these last years, where the filtering process depends on the local properties of the image. This concept was first introduced by P&M [2] for image enhancement and edge detection. Anisotropic diffusion is a process in which the denoising is guided by the spatial derivative; therefore-there is no smoothing across edges. For this reason, Persona and Malik replaced the isotropic conductivity by a scalar function which is defined for each pixel:

$$\begin{cases} \dfrac{\partial I(x,y,t)}{\partial t} = div\left[c\left(\|\nabla I(x,y,t)\|\right) \cdot \nabla I(x,y,t)\right] \\ I(x,y,0) = I_0. \end{cases} \tag{2}$$

Assuming that $I(x, y)$ is the intensity of a pixel $p(x, y)$ at the location (x, y) in an image I, which is defined as a limited application of $\Omega \subset \Re^2 \rightarrow \Re$, Ω is the image field and $I_0(x, y) = I_0$ denotes the distorted image, div is the divergence operator, The gradient magnitude is presented by $\|\nabla I\|$, c is called the conductance function which has a decreasing feature of the gradient magnitude. Accordingly, this function is designed to reduce the diffusion coefficients under the edges of an image so that $\lim_{x\to\infty} c(x) = 0$ and keep it in areas with low gradient where the diffusion is maximal such that $\lim_{x\to 0} c(x) = 1$. Originally, two functions were proposed:

$$c_1\left(\|\nabla x\|\right) = \frac{1}{1 + \left(\frac{\|\nabla x\|}{k}\right)^2} \tag{3}$$

and

$$c_2(\|\nabla x\|) = \exp\left[-\left(\frac{\|\nabla x\|}{k}\right)^2\right] \tag{4}$$

where k is a positive threshold parameter. By these choices of c_1 and c_2, Perona and Malik argued that their anisotropic diffusion model has virtuous denoising results and efficient edge protection. The flow function ϕ is expressed as follows:

$$\phi(x) = x * c(x). \tag{5}$$

This function increases where it smoothes the uniform zones and then decreases where the diffusion is stopped to preserve the edges and the contents of the images. The flow function is proportionally rated by k.

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