# Global existence of classical solutions to the cross-diffusion three-species model with prey-taxis☆

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## A R T I C L E   I N F O

## A B S T R A C T

This paper deals with the cross-diffusion three-species model with prey-taxis incorporating Beddington–DeAngelis response under homogeneous Neumann boundary condition. In addition to random diffusion, the predators and preys have cross-diffusion and prey-taxis during the process of interaction. By applying the contraction mapping principle, the Hölder continuity, the parabolic Schauder estimates and parabolic $L_p$ estimates, we prove that there exists a unique global classical solution of this system.

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## 1. Introduction

The term self-diffusion implies the movement of individuals from a higher to lower concentration region. Cross-diffusion describes the flux of species caused from the mutual interferences which indicates the movement of the species in the direction of lower concentration of another species when the cross-diffusion coefficients are positive and the tendency of the population in the direction of higher concentration of another species when the cross-diffusion coefficients are negative. In paper [1], the authors put forward a general cross-diffusion–reaction species model by numerical simulation. Taxis is defined as the stimulus-induced movement of animal [2,3] and prey-taxis is defined as the movement of predators controlled by prey density. In paper [4], Sapoukhina et al. assume that the directed movement of predator density is determined by the acceleration, which is proportional to the prey density gradient, or, in general, to the gradient of some stimulus. Several works measuring characteristics of individual movement verify the basic hypothesis about the dependence of acceleration on a stimulus [5,6].

If mutual interference among predators and the handling time of each prey are both considered, then a more reasonable response function is $\phi(u) = \frac{u}{1+Bu+Cv}$ (see [7]), called a Beddington–DeAngelis response. This response function is derived mechanistically by considerations of time utilization (see [8,9]) or spatial limits on predation (see [10]). The term $Cv$ reflects mutual interference among predators, while the term $Bu$ is regarded as reflecting limits on predation to handle an item of prey.

The existence of the global solutions is a central topic in the partial differential equations. In paper [11], the author proved existence and uniqueness of classical solutions and He Xiao et al. verified furthermore that these global classical solutions are globally bounded (see [12]) for the two-component system with prey-taxis. In paper [13], the authors investigated the existence of classical solution in $C^{2+\alpha,1+\alpha/2} \times C^{2+\alpha,1+\alpha/2}(0 < \alpha < 1)$ to the two-component cross-diffusion system. In article [14], Wang Xiaoli et al. considered global bifurcation of solutions for a predator–prey model with prey-taxis.

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The authors in [15] investigated gradient estimates and global existence of smooth solutions to a cross-diffusion system. However, to our knowledge, there is no paper investigating the reaction–diffusion systems both including cross-diffusion and prey-taxis.

Our objective of this paper is to prove the predator–prey system both including cross-diffusion and prey-taxis with Beddington–DeAngelis functional response as follows:

$$
\begin{aligned}
&\frac{\partial u_1}{\partial t} - \Delta[(d_1 + d_2/(1 + \rho v))u_1] = u_1\left(-a + \frac{A_1 v}{1 + B_1 u_1 + C_1 v}\right) \quad \text{in } (0, T) \times \Omega, \\
&\frac{\partial u_2}{\partial t} - d_3 \Delta u_2 + \nabla \cdot (u_2 \chi(u_2)\nabla v) = u_2\left(-b + \frac{A_2 v}{1 + B_2 u_2 + C_2 v}\right) \quad \text{in } (0, T) \times \Omega, \\
&\frac{\partial v}{\partial t} - \Delta[(d_4 + d_5 v)v] = rv(1 - v/K) - \frac{A_1 u_1 v}{1 + B_1 u_1 + C_1 v} - \frac{A_2 u_2 v}{1 + B_2 u_2 + C_2 v} \quad \text{in } (0, T) \times \Omega, \\
&\frac{\partial u_1}{\partial \nu} = \frac{\partial u_2}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on } (0, T) \times \partial\Omega, \\
&(u_1(0, x), u_2(0, x), v(0, x)) = (u_{10}(x), u_{20}(x), v_0(x)) \geq (0, 0) \quad \text{in } \Omega,
\end{aligned}
\tag{1.1}
$$

where $\Omega$ is a bounded domain with a smooth boundary $\partial\Omega$ in $R^N (N = 1, 2, 3)$; $u_1, u_2$ and $v$, respectively, represent the density of the predator and prey; the positive constants $d_1, d_2, d_3, d_4$ and $d_5$ are the diffusion coefficients of the corresponding species respectively; the positive constants $a, b, K, r$ represent the death rate of the predator, the carrying capacity of prey, the prey intrinsic growth rate, respectively.

In diffusion terms, $d_1, d_3$ and $d_4$ represent the natural dispersive force of movement of population, while $d_2$ describes the mutual interferences between predators and preys, $d_5$ describes the mutual interferences of preys. $d_2$ and $d_5$ are also called cross-diffusion pressure and self-diffusion pressure respectively. The model (1.1) means that, in addition to the natural dispersive force, the diffusion also depends on the pressure of population from other species and prey-taxis. The diffusion given rise by the inter-pressure is cross-diffusion. The part $\nabla \cdot (u_2 \chi(u_2)\nabla v)$ of the flux to the predator $u_2$ is directed toward the increasing population density of $v$, which indicates that the predators move in the direction of higher concentration of the preys. The term self-diffusion implies the movement of individuals from a higher to lower concentration region due to the intra-pressure of the same species.

In the system (1.1), the predator $u_1$ diffuses with flux

$$
\begin{aligned}
J &= -\nabla[(d_1 + d_2/(1 + \rho v))u_1] \\
&= -(d_1 + d_2/(1 + \rho v))\nabla u_1 + (d_2 \rho u_1/(1 + \rho v)^2)\nabla v.
\end{aligned}
$$

The part $-(d_1 + d_2/(1 + \rho v))\nabla u_1$ of the flux is directed toward the decreasing density of the predators, which implies that preys run away from the predators to avoid being catched. The part $(d_2 \rho u_1/(1 + \rho v)^2)\nabla v$ of the flux is directed toward the increasing density of the prey, which implies that predators move toward the preys to catch.

Throughout this paper we assume that

$$
\begin{aligned}
&v_0 \leq K, \qquad \partial\Omega \in C^{2+\alpha}, \qquad u_{10}(x), u_{20}(x), v_0(x) \in C^{2+\alpha}, \\
&\frac{\partial u_{10}}{\partial \nu} = \frac{\partial u_{20}}{\partial \nu} = \frac{\partial v_0}{\partial \nu} = 0, \quad \text{on } \partial\Omega,
\end{aligned}
\tag{1.2}
$$

where $0 < \alpha < 1$.

For the convenience, we denote $Q_T = (0, T) \times \Omega$ and $\partial\Omega_T = (0, T) \times \partial\Omega$.

The main result of this paper is as follows:

**Theorem 1.1.** *Under assumptions* (1.2), *for any given* $T > 0$, *there exists a unique solution* $U = (u_1, u_2, v) \in C^{2+\alpha, 1+\frac{\alpha}{2}}(Q_T)$ *of the system* (1.1). *Moreover,*

$$
u_1(x, t) \geq 0, \qquad u_2(x, t) \geq 0, \qquad 0 \leq v \leq K
$$

*for any* $x \in \Omega$ *and* $t > 0$.

This paper is organized into three sections. In Section 2, we prove local existence and uniqueness to system (1.1). In Section 3, we make a priori estimates. In Section 4, we prove global existence to system (1.1).

## 2. Local existence and uniqueness of solutions

In this section, we shall prove the existence of the unique local solution to the system (1.1). For the convenience, in the following we denote various constants which are independent of $T$ by $N_0$, and we also denote various constants which depend on $T$ by $N$.