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Study on fractional non-autonomous evolution equations with delay

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ABSTRACT

In this paper, we deal with a class of nonlinear time fractional non-autonomous evolution equations with delay by introducing the operators $\psi(t,s)$, $\varphi(t,\eta)$ and $U(t)$, which are generated by the operator $-A(t)$ and probability density function. The definition of mild solutions for studied problem was given based on these operators. Combining the techniques of fractional calculus, operator semigroups, measure of noncompactness and fixed point theorem with respect to k-set-contractive, we obtain new existence result of mild solutions with the assumptions that the nonlinear term satisfies some growth condition and noncompactness measure condition and the closed linear operator $-A(t)$ generates an analytic semigroup for every $t > 0$. The results obtained in this paper improve and extend some related conclusions on this topic. At last, by utilizing the abstract result obtained in this paper, the existence of mild solutions for a class of nonlinear time fractional reaction–diffusion equation introduced in Ouyang (2011) is obtained.

1. Introduction

In this paper, we investigate the existence of mild solutions for initial value problem to the following nonlinear time fractional non-autonomous evolution equations with delay in Banach space E

$$\begin{cases} {}^C D_t^\alpha u(t) + A(t)u(t) = f(t, u(\tau_1(t)), \dots, u(\tau_m(t))), & t \in J, \\ u(0) = u_0, \end{cases} \tag{1.1}$$

where ${}^C D_t^\alpha$ is the standard Caputo's fractional time derivative of order $0 < \alpha \le 1$, $J = [0, T_0]$, $T_0 > 0$ is a constant, $\{A(t)\}_{t\in J}$ is a family of closed linear operators defined on a dense domain $D(A)$ in Banach space E into E such that $D(A)$ is independent of t, m is a positive integer number, $\tau_k : J \to J$ are continuous functions such that $0 \le \tau_k(t) \le t$ for $k = 1, 2, \dots, m$, $f : J \times E^m \to E$ is continuous and $u_0 \in E$.

In recent years, fractional calculus has attracted many physicists, mathematicians, engineers and notable contributions have been made to both theory and applications of fractional differential equations. The most important advantage of fractional derivatives compared with integer derivatives is that it describe the property of memory and heredity of various materials and processes. In the mathematical modeling of physical phenomena, fractional differential equations are found to be better tools than their corresponding integer-order counterparts, for example, the description of anomalous diffusion via such equations leads to more informative and interesting model [1].

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One of the branches of fractional calculus is the theory of fractional evolution equations. Since fractional order semilinear evolution equations are abstract formulations for many problems arising in engineering and physics, fractional evolution equations have attracted increasing attention in recent years, see [2–12] and the references therein. Among the previous researches, most of researchers focus on the case that the differential operators in the main parts are independent of time t, which means that the problems under considerations are autonomous. However, when treating some parabolic evolution equations, it is usually assumed that the partial differential operators depend on time t on account of this class of operators appears frequently in the applications, for the details please see [13]. Therefore, it is interesting and significant to investigate fractional non-autonomous evolution equations, i.e., the differential operators in the main parts of the considered problems are dependent of time t.

In 2004, El-Borai [14] investigated the existence and continuous dependence of fundamental solutions for a class of linear fractional non-autonomous evolution equations. Later, in 2011, by using Lebesgue dominated convergence theorem, Leray–Schauder fixed point theorem and Banach contraction mapping theorem, Ouyang [15] give some sufficient conditions for the existence of solutions for the initial boundary value problem to the following nonlinear time fractional reaction–diffusion equation with delay

$$\begin{cases} {}^{C}D_t^{\alpha}u(x,t) - a(t)\Delta u(x,t) = f(t, u(x,\tau_1(t)), \ldots, u(x,\tau_m(t))), & t \in J, \\ u(x,t) = 0, \quad x \in \partial\Omega, \quad t \in J, \\ u(x,0) = \varphi(x), \quad x \in \Omega, \end{cases} \tag{1.2}$$

where Δ is the Laplace operator, $\Omega \subset \mathbb{R}^m$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$, $f : J \times \mathbb{R}^m \to \mathbb{R}$ is a nonlinear function and $\varphi \in L^2(\Omega)$. Recently, by using a different method with which used in [15], Zhu, Liu and Wu [16] also studied the existence and uniqueness of global mild solutions for the problem (1.2). In [16], the authors firstly transfer the initial boundary value problem of time fractional reaction–diffusion equation (1.2) into the abstract form of time fractional non-autonomous evolution equations (1.1) in Banach space $L^2(\Omega)$, then utilizing measure of noncompactness, the theory of resolvent operators, the fixed point theorem and the Banach contraction mapping principle to obtain the existence and uniqueness of the global mild solutions for the problem (1.2). However, we should mention that the definition of the mild solutions for the transformed abstract fractional non-autonomous evolution equations (1.1) is not right. In fact, the definition in [16] (see [16, Definition 2.4]) is only valid for the case $\alpha = 1$, i.e. the linear operator $-A(t)$ generates an evolution system $U(t,s)$. For the details, please see Henry [17] and Pazy [18].

Based on the above mentioned aspects, it is natural to investigate the initial value problem of nonlinear time fractional non-autonomous evolution equations (1.1) for a general case $0 < \alpha \le 1$. In the present paper, our purpose is to introduce three operators generated by the operator $-A(t)$ and probability density function, and then give the proper definition of mild solutions for the initial value problem of nonlinear time fractional non-autonomous evolution equations (1.1), which plays a key role in our discussion. With the aid of this definition and the properties of these operators, the existence of mild solution for the initial value problem of nonlinear time fractional non-autonomous evolution equations (1.1) is obtained under certain assumptions on the nonlinear term f, by using appropriate fractional calculus and fixed point theorem with respect to k-set-contractive. At last, by utilizing the abstract theorem obtained in this paper, as an example, we also obtained the existence of mild solutions for the nonlinear time fractional reaction–diffusion equation (1.2) introduced by Ouyang [15] and Zhu, Liu and Wu [16].

The rest of this paper is organized as follows. In Section 2, we present some preliminaries on fractional calculus, the definitions of the operators $\psi(t,s)$, $\varphi(t,\eta)$, $U(t)$ and their properties, and the results about Kuratowski measure of noncompactness, which will be used in the proof of our main result. Section 4 states and prove the existence of mild solutions for the initial value problem of time fractional non-autonomous evolution equations with delay (1.1). In Section 4, an example about the nonlinear time fractional reaction–diffusion equation with delay is illustrated.

2. Preliminaries

Throughout this work, we set $J = [0, T_0]$, where $T_0 > 0$ is a constant. Let E be a Banach space with norm $\|\cdot\|$. We denote by $C(J,E)$ the Banach space of all continuous functions from interval J into E equipped with the supremum norm $\|u\|_C = \sup_{t\in J}\|u(t)\|$, and by $\mathcal{L}(E)$ the Banach space of all linear and bounded operators in E endowed with the topology defined by operator norm. Let $L^1(J,E)$ be the Banach space of all E-value Bochner integrable functions defined on J with the norm $\|u\|_1 = \int_0^{T_0}\|u(t)\|dt$.

At first, we recall the definitions of the Riemann–Liouville integral and Caputo derivative of fractional order.

Definition 2.1 ([19]). The fractional integral of order $\alpha > 0$ with the lower limit zero for a function $f \in L^1([0,+\infty), \mathbb{R})$ is defined as

$$I_t^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)ds.$$

Here and elsewhere Γ denotes the Gamma function.

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