ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Free vibration analysis of embedded single-layered nanoplates and graphene sheets by using the multiple time scale method

Chih-Ping Wu*, Wei-Chen Li

Department of Civil Engineering, National Cheng Kung University, Tainan 70101, Taiwan, Republic of China

ARTICLE INFO

Article history:
Received 6 July 2016
Received in revised form 21 December 2016
Accepted 16 January 2017
Available online xxxx

Keywords:
Graphene sheets
Multiple time scale methods
Nanoplates
Nonlocal
Pasternak's foundation
Vibration

ABSTRACT

An asymptotic approach using the Eringen nonlocal elasticity theory and multiple time scale method is developed for the three-dimensional (3D) free vibration analysis of simply-supported, single-layered nanoplates and graphene sheets (GSs) embedded in an elastic medium. In the formulation, the small length scale effect is first introduced to the nonlocal constitutive equations by using a nonlocal parameter, then the mathematical processes of nondimensionalization, asymptotic expansion and successive integration are performed, and finally recurrent sets of motion equations for various order problems are obtained. The interactions between the nanoplates (or GSs) and their surrounding medium are modeled as a two-parameter Pasternak foundation. Nonlocal classical plate theory (CPT) is derived as a first-order approximation of the 3D nonlocal elasticity theory, and the motion equations for higher-order problems retain the same differential operators as those of nonlocal CPT, although with different nonhomogeneous terms. Some 3D nonlocal elasticity solutions of the natural frequency parameters of nanoplates (or GSs) with and without being embedded in the elastic medium and their corresponding through-thickness distributions of modal field variables are given to demonstrate the performance of the 3D asymptotic nonlocal elasticity theory.

1. Introduction

In recent years, the fields of nanoscience and nanotechnology have developed significantly, since some nanostructured elements, such as the beam-like (or circular hollow cylinder-like) carbon nanotubes (CNTs) [1] and plate-like graphene sheets (GSs) [2], were first discovered. Due to their excellent mechanical, chemical, thermal and electrical material properties, CNTs and GSs have thus been introduced in a variety of potential applications to both micro- and nano-electro-mechanical systems (MEMs and NEMs), such as communication, machinery, information, and biological technologies [3–6]. They have also been used as the embedded reinforcements to enhance the performance of laminated composite macrostructures [7–9]. A comprehensive literature survey with regard to the application of nonlocal elastic models in the modeling of CNTs and GSs has been undertaken, and can be found in the literature [10–15].

Because CNTs and GSs have being widely used in various advanced industries, their structural analysis with and without being embedded in an elastic medium have thus attracted considerable attention in order to extend their lifetimes and

* Corresponding author. Fax: +886 6 2370804.
E-mail address: cpwu@mail.ncku.edu.tw (C.-P. Wu).

http://dx.doi.org/10.1016/j.camwa.2017.01.014
0898-1221/

Please cite this article in press as: C.-P. Wu, W.-C. Li, Free vibration analysis of embedded single-layered nanoplates and graphene sheets by using the multiple time scale method, Computers and Mathematics with Applications (2017), http://dx.doi.org/10.1016/j.camwa.2017.01.014

enhance their performance, especially with regard to their dynamic characteristics. This paper thus focuses on a literature survey related to the free vibration analysis of embedded nanoplates and GSs using the Eringen nonlocal elasticity theory (ENET) [16–18], which is more computationally efficient than the other two branches for the analysis of nanostructures, namely the atomistic [19,20] and hybrid atomistic–continuum mechanics approaches [21,22].

Most of the studies in the open literature with regard to the free vibration analysis of nanoplates and GSs with and without being embedded in an elastic medium are based on two-dimensional (2D) nonlocal plate theories, which are reformulated using the ENET. Aghababaei and Reddy [23] reformulated the local higher-order shear deformation theory (HSDT) for laminated composite plates [24,25] to analyze the bending behavior and vibration response of nanoplates using the ENET, in which the small scale and shear deformation effects were captured in their nonlocal formulation. In conjunction with the differential quadrature (DQ) method and ENET, the local classical plate theory (CPT) was reformulated by Pradhan and Kumar [26,27] for the linear vibration of isolated and embedded single-layered orthotropic GS with various boundary conditions. Their work considered the effects of the nonlocal parameter, aspect ratio, stiffness of surrounding elastic medium and boundary conditions on the vibration characteristics of nanoplates and GSs. The nonlinear vibration counterparts of the above-mentioned nanoplates and GSs were investigated by Shen et al. [28] and Shen [29], in which the von Karman-type kinematic nonlinearity and thermal effect were taken into account. The nonlocal CPT combined with the Rayleigh–Ritz and finite strip methods was used to examine the free vibration of rectangular nanoplates by Chakraverty and Behera [30]. Based on the nonlocal first-order shear deformation theory (FSDT), Ansari et al. [31,32] examined the free vibration behavior of embedded, multi-layered GSs with various boundary conditions, in which the differential quadrature (DQ) method was used for the numerical computation and the van der Waals forces between adjacent and non-adjacent layers of multilayered GSs were considered. Based on the nonlocal CPT and FSDT, Liew et al. [33] and Pradhan and Phadikar [34] showed the Navier's solutions of the natural frequency parameters of simply-supported, multilayered GSs embedded in an elastic medium, in which the effects of the nonlocal parameter, aspect ratio, and material properties on the frequency parameters were investigated. The issues were also examined for single-layered nanoplates and GSs by Thai et al. [35] using the nonlocal sinusoidal plate theory, and Malekzadeh and Shojaee [36] using the nonlocal two-variable refined plate theory. In order to extend the scope of application, Wang and Wang [37] thus developed a continuum finite element model on the basis of nonlocal CPT and FSDT for the bending and vibration analyses of nanoplates and GSs.

However, there are rarely few articles that carry out the three-dimensional (3D) free vibration analysis of simply-supported, nanoplates and GSs, as compared to the 2D analysis of these. In conjunction with the state space and DQ methods, Brischetto et al. [38] applied the 3D and 2D refined shell models to the 3D free vibration analysis of single- and double-walled carbon nanotubes (CNTs), in which the van der Waals interactions between the two cylinders were included and the natural frequency parameters and their corresponding vibration modes were presented. The state space analytical solutions of the 3D free vibration of simply-supported, single-walled CNTs were presented by Brischetto et al. [39]. Ansari et al. [40] studied the 3D bending and vibration analyses of functionally graded (FG) nanoplates with various boundary conditions, as well as with resting on the elastic medium, in which Hamilton's principle was used to derive the motion equations of the nanoplate, the material properties of which were assumed to vary through the thickness coordinate with power-law and exponential distributions.

To the best of the authors' knowledge, the perturbation method [41] has never been applied to the 3D mechanical analysis of nanoplates and GSs, even though it has been successfully applied to that of macrostructures, such as laminated composite structures [42–47] and FG elastic/piezoelectric ones [48–52]. Within the 3D nonlocal elasticity theory, we thus developed an asymptotic theory for the 3D free vibration analysis of simply-supported, single-layered rectangular nanoplates and GSs embedded in the elastic medium by using the method of multiple time scales and Eringen's nonlocal elasticity theory. In the formulation, we first reduce the fifteen partially differential equations (PDEs) of the 3D nonlocal elasticity theory to six PDEs in terms of six primary variables, which are three displacement components and three transverse shear and normal stress ones. By asymptotically expanding the primary variables as a power series of a small geometric parameter and the time variable as multiple time scale parameters, we finally obtain the recurrent sets of nonlocal motion equations for various order problems. The nonlocal CPT is derived as a first-order approximation of the 3D nonlocal elasticity theory, and the nonlocal motion equations for higher-order problems retain the same differential operators as those of the nonlocal CPT, although with different nonhomogeneous terms. We can obtain the Navier solutions of the leading-order problem by using the double Fourier series expansion method. By satisfying the solvability and normality conditions, the secular terms of higher-order problems can be removed, and the unique modal field variables can be obtained. The higher-order modifications can then be determined in a systematic manner. The effects of the nonlocal parameter, aspect ratio, axial stiffness and shear modulus of the medium on the natural frequency parameters and their associated modal field variables for the nanoplates and GSs are also examined.

The 3D asymptotic nonlocal theory differs from the above-mentioned 2D nonlocal classical and refined theories, such as nonlocal CPT, nonlocal FSDT and nonlocal HSDT. In the former, without kinetic and kinematic assumptions as *a prior*, the nonlocal CPT was derived as the leading-order approximation, and its accuracy can be improved order-by-order by solving the governing equations of the higher-order problems without reformulation. In the latter, they need to assume a specific displacement field through the thickness direction of the nanoplate before starting the formulation. The accuracy of these 2D nonlocal classical and refined theories is difficult to assess. If it is necessary to enhance the accuracy of these theories, the entire formulation will be rewritten by assuming a new displacement field.