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Variational approach and exact solutions for a generalized coupled Zakharov–Kuznetsov system

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ABSTRACT

In the present paper, we obtain a variational principle for a generalized coupled Zakharov–Kuznetsov system, which does not admit any Lagrangian formulation in its present form. The eminent Noether's theorem will then be employed to compensate for this approach. In addition, exact solutions will be constructed for the generalized coupled Zakharov–Kuznetsov system using the Kudryashov method and the Jacobi elliptic function method.

1. Introduction

In [1], the authors derived the generalized coupled Zakharov–Kuznetsov system

$$\begin{cases} u_t + u_{xxx} + u_{yyx} - 6uu_x - v_x = 0, \\ v_t + \delta v_{xxx} + \lambda v_{yyx} + \eta v_x - 6\mu v v_x - \beta u_x = 0. \end{cases} \tag{1}$$

The coupled Zakharov–Kuznetsov system describes two interacting weakly nonlinear waves in anisotropic back-ground stratified fluid flows. Here x and y represent the propagation and transverses coordinates respectively, while η is the group-velocity shift between the coupled models, δ, λ are the relative longitudinal and transverse dispersion coefficients, and μ, β represent the relative nonlinear and coupling coefficients. It is easy to see that if the transverse variation ($u_y = v_y = 0$), the coupled Zakharov–Kuznetsov system reduces to a family of Korteweg–de Vries equations [1], which describe the interaction of the nonlinear long waves in various fluid flows. Indeed, Zakharov–Kuznetsov system has been studied by other researchers using different approaches. For example, the polynomial expansion method, extended Jacobi elliptic function expansion method and the modified extended tanh method were used to find solitary wave solutions, periodic solutions and rational type solutions [2–4]. See also [5,6].

In this paper, we will work with a slight modification of the generalized coupled Zakharov–Kuznetsov system (1), namely

$$\begin{cases} u_t + u_{xxx} + u_{yyx} - 6uu_x - v_x = 0, \\ v_t + \delta v_{xxx} + \lambda v_{yyx} + \eta v_x - 6\mu v v_x - u_x = 0. \end{cases} \tag{2}$$

The objective of the present study is to construct conservation laws for system (2) using Noether's approach. Thereafter, we focus our investigations on the derivation of exact solutions for the generalized coupled Zakharov–Kuznetsov system (2) by invoking the Kudryashov method and the Jacobi elliptic function method.

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2. Fundamental relations

We briefly present some fundamental notations and definitions to be used in Section 3. For details, we direct the diligent reader to [7–11].

Let us consider an nth-order system of k partial differential equations with $x = (x^1, x^2, \ldots, x^m)$ independent variables and $u = (u^1, u^2, \ldots, u^q)$ dependent variables, namely

$$E^\alpha = (x, u, u_{(1)}, u_{(2)}, u_{(3)}, \ldots, u_{(n)}) = 0, \quad \alpha = 1, \ldots, k, \tag{3}$$

where $u_{(i)}$ denote the collection of ith-order of partial derivatives with $u_i^\alpha = D_i(u^\alpha), u_{ij}^\alpha = D_jD_i(u^\alpha), \ldots$, where the total differential operator is defined by

$$D_i = \frac{\partial}{\partial x^i} + u_i^\alpha \frac{\partial}{\partial x^i} + u_{ij}^\alpha \frac{\partial}{\partial u_j^\alpha} + \cdots, \quad i = 1, \ldots, m. \tag{4}$$

Consider the vector field

$$X = \xi^i \frac{\partial}{\partial x^i} + \eta^\alpha \frac{\partial}{\partial u^\alpha} + \sum_{s\geq 1} \zeta^\alpha_{i_1\ldots i_s} \frac{\partial}{\partial u^\alpha_{i_1\ldots i_s}}, \tag{5}$$

where $\zeta^\alpha_{i_1\ldots i_s}$ are defined in [12] and references therein.

The vector field (5) in characteristic form is

$$X = \xi^i \frac{\partial}{\partial x^i} + W^\alpha \frac{\partial}{\partial u^\alpha} + D_i(W^\alpha) \frac{\partial}{\partial u_i^\alpha} + D_iD_j(W^\alpha) \frac{\partial}{\partial u_{ij}^\alpha} + \cdots, \tag{6}$$

where W^α is the Lie characteristic function.

The Noether operators corresponding to the vector field X are defined by

$$N^i = \xi^i + W^\alpha \frac{\delta}{\delta u_i^\alpha} + \sum_{s\geq 1} D_i \ldots D_{i_s}(W^\alpha) \frac{\delta}{\delta u^\alpha_{ij\ldots j_s}}, \tag{7}$$

where

$$\frac{\delta}{\delta u_i^\alpha} = \frac{\partial}{\partial u_i^\alpha} + \sum_{s\geq 1} D_{j_1} \ldots D_{j_{i_s}}(W^\alpha) \frac{\delta}{\delta u^\alpha_{ij\ldots j_s}}, \quad i = 1, \ldots, m, \ \alpha = 1, \ldots, k \tag{8}$$

is the Euler–Lagrange operator.

2.1. Euler–Lagrange equation

A function $L(x, u, u_{(1)}, u_{(2)}, u_{(3)}, \ldots, u_{(s)}), s \leq k$ is said to be a Lagrangian of system (3) if the Euler–Lagrange equation

$$\frac{\delta L}{\delta u^\alpha} = 0, \quad \alpha = 1, \ldots, k. \tag{9}$$

2.2. Noether symmetry operator

The vector field X is called a Noether point symmetry operator corresponding to the Lagrangian $L(x, u, u_{(1)}, u_{(2)}, u_{(3)}, \ldots, u_{(k-1)})$ if there exists the point-dependent gauge terms $A = (A^1, A^2, A^3, \ldots, A^m)$ such that the Killing-type equation

$$X(L) + \{D_i(\xi^i)\}L = D_i(A^i) \tag{10}$$

holds.

We now state the prominent Noether theorem.

Theorem 1 (*Noether [7]*). *If X is a Noether point symmetry operator associated with the Lagrangian $L(x, u, u_{(1)}, u_{(2)}, u_{(3)}, \ldots, u_{(k-1)})$ of Eq. (3), there corresponds a vector $T = (T^1, T^2, \ldots, T^m)$ where T^i is defined by*

$$T^i = A^i - N^iL \tag{11}$$

is a conserved vector for Eq. (3) associated with the operator X.