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Bandwidth selection of nonparametric threshold estimator in jump–diffusion models

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ABSTRACT

In this paper, we first study the asymptotic properties of nonparametric threshold estimator of diffusion function in jump–diffusion models. Two-dimensional asymptotics in both the time span and the sampling interval are provided. Our results provide a precise characterization for the limiting distributions of nonparametric threshold estimator. At last, we present the determination of optimal bandwidth for nonparametric threshold estimator.

1. Introduction

In this paper, we study the asymptotic properties of nonparametric kernel threshold estimator of jump–diffusion models. We describe the evolution of a economic variable, e.g. an interest rate or a logarithmic asset price, through the following stochastic differential equation (SDE):

$$dX_t = \mu_t dt + \sigma(X_{t-})dW_t + dJ_t \tag{1.1}$$

where W is a standard Brownian motion, and J is a pure-jump process, $X_{t-} = \lim_{\delta\to 0+} X_{t-\delta}$.

In the past decades, the nonparametric estimations of coefficients in diffusion and jump–diffusion models were wildly studied, for example, Florens-Zmirou [1], Bandi and Phillips [2], Bandi and Nguyen [3], Mancini and Renò [4], Muhammad et al. [5] and so on. Usually, the sample is obtained at equidistant intervals of length $\triangle$ over a time span T. The conventional asymptotics of nonparametric coefficients rely on the sample size $n = T/\triangle$.

Recently, Aït-Sahalia and Park [6] consider the case where both $\triangle \to 0$ and $T \to \infty$, the so-called two-dimensional asymptotics, for diffusion models. Two-dimensional asymptotics allow the time span T, the sampling interval $\triangle$ and the bandwidth h to change simultaneously. Two-dimensional asymptotics have several distinguishing features, for example, providing a single framework to unify the limit theories for stationary and nonstationary diffusion, analyzing the performance of estimators under various sampling schemes, providing a characterization of the bias and variance for nonparametric estimators and so on. More important, since bandwidth h is generally obtained as a function of T and $\triangle$, two-dimensional asymptotics can determine an optimal bandwidth.

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Aït-Sahalia and Park [6] provide framework and approach in two-dimensional asymptotics for diffusion models. We extend their work to jump–diffusion model.

The statistical inference for jump–diffusion model is widely studied, even, some more general models, for example, semimartingale with jump, are discussed deeply, such as Aït-Sahalia and Jacod [7–9], Jing et al. [10], Li [11], Todorov et al. [12], Lin and Wang [13] and so on. We prefer a systematic monograph, Jacod and Protter [14].

In this paper, our model (1.1) has level-dependent drift and volatility, we employ nonparametric smooth fitting to study this model. Our framework and approach are most closely related to the work of Mancini and Renò [4], who use the threshold kernel estimation and obtain the asymptotic properties. We obtain a precise characterization for the limiting distributions of nonparametric threshold estimator. At last, we present the determination of optimal bandwidths for these estimators.

The paper is structured as follows. In Section 2, we set up the model and present some preliminaries on jump–diffusion model. In Section 3, we concentrate on asymptotic properties of local time estimator for jump–diffusion models, which are very useful in our study. A precise characterization for the limiting distribution is given in the last section, and we present the determination of optimal bandwidth for this estimator in this section.

2. Model setup

In this section, we set up the jump–diffusion model and the assumptions. We focus on a filtered space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$, satisfying the usual conditions (Jacod and Shiryaev [15]).

We model the evolution of an economic variable, a price, by the solution of

$$dX_t = \mu(X_{t-})dt + \sigma(X_{t-})dW_t + dJ_t$$

where W is standard Brownian motion, $\{J_t, t \geq 0\}$ is a finite activity pure jump semimartingale, which is independent of W.

In this paper, we assume that

$$J_t = \sum_{l=1}^{N_t} \gamma_l,$$

where N_t is a compound Poisson process, γ_l is $i.i.d.$ random variable. We will study the more complex sense in the future. In the whole paper, we assume that the model is recurrence, which allow the model to be stationary or non-stationary.

We assume that X is equidistant observed over time $[0, T]$ and the sampling interval is Δ. Sample size is $n = \frac{T}{\Delta}$. Relative to the time span T, when sampling interval $\Delta \to 0$ sufficiently fast, sample size $n \to \infty$. Denote observations as $X_i = X_{i\Delta}$, $i = 1, \ldots, n$ and the incremental of X in time i is $\Delta_i X = X_i - X_{i-1}$.

If we consider the estimation of diffusion model

$$dX_t = \mu_t dt + \sigma(X_{t-})dW_t, \tag{2.1}$$

we can use the kernel estimation

$$\check{\sigma}_n^2(x) = \frac{\sum_{i=1}^{n} K\left(\frac{X_{i-1}-x}{h_n}\right)\frac{(\Delta_i X)^2}{\Delta}}{\sum_{i=1}^{n} K\left(\frac{X_{i-1}-x}{h_n}\right)}$$

to estimate the diffusion function $\sigma^2(x)$, where $K(x)$ is kernel function. However, when we consider the model (1.1), $\check{\sigma}_n^2(x)$ cannot estimate the diffusion function $\sigma^2(x)$ directly, since the impact of the jump part.

Mancini and Renò [4] introduce a nonparametric threshold estimator to estimate the diffusion function $\sigma^2(x)$ directly:

$$\hat{\sigma}_n^2(x) = \frac{\sum_{i=1}^{n} K\left(\frac{X_{i-1}-x}{h_n}\right)\frac{(\Delta_i X)^2}{\Delta} 1_{\{(\Delta_i X)^2 \leq \theta(\Delta)\}}}{\sum_{i=1}^{n} K\left(\frac{X_{i-1}-x}{h_n}\right)}$$

where $\theta(x)$ is the threshold function to eliminate the impact of the jump part.

Mancini and Renò [4] obtained the asymptotic normality of $\hat{\sigma}_n^2(x)$, but they did not present the optimal bandwidth for nonparametric threshold estimator. We drive a precise characterization for the limiting distribution in this paper, and present the determination of optimal bandwidth.

We assume that the model (1.1) is recurrence, thus the density function of X_t may be not existed. We use the local time of process X to replace the density function, which is very useful in the asymptotic study of kernel estimation. The local time of processes X is defined as follows:

$$L(T, x) = \lim_{\varepsilon\to 0} \frac{1}{2\varepsilon}\int_0^T 1_{\{|X_{s-}-x|<\varepsilon\}}\,\mathrm{d}s.$$