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Periodic attractor for reaction–diffusion high-order Hopfield neural networks with time-varying delays

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ABSTRACT

This paper is concerned with a class of reaction–diffusion high-order Hopfield neural networks with time-varying delays subject to the Dirichlet boundary condition in a bounded domain. Easily verifiable delay-independent criteria are established to ensure the existence of periodic mild solutions, and the global exponential stability of the periodic mild solutions is also discussed by using the exponential dissipation property of semigroup of operators. The obtained results are easy to check and they effectually complement previously known results. A numerical example is given to show the effectiveness of theoretical results.

1. Introduction

In recent years, different types of neural network models have been studied extensively and have been applied widely in many areas such as combinatorial optimization, signal processing, pattern recognition, speed detection of moving objects, optimization and associative memories, see [1–3]. In particular, high-order Hopfield neural networks, as an important class of dynamical systems, have been the object of intensive analysis by many authors in both theory and application due to the fact that high-order neural networks can be with impressive computational, learning, and storage capabilities [4], and have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than the low-order neural networks [5]. Therefore, much research attention has been given to the study of the dynamical behaviors for high-order neural networks, see, e.g., [6–11] and the references therein.

As is well known, in the modeling of the artificial neural networks or biological neuron networks, it is sometimes necessary to take account of time delays inherent in the dynamic phenomena because of the finite processing speed of information, for instance, the finite axonal propagation speed from soma to synapses, the diffusion of chemical across the synapses, the postsynaptic potential integration of membrane potential at the neuronal cell body and dendrites. Furthermore, in the electronic implementation of artificial neural networks, the time delays are omnipresent in the communication and response of neurons owing to the finite switching speed of amplifiers [12]. Moreover, to process moving images, one must introduce time delays in the signals transmitted among the cells [13], and time delays may lead to instability, divergence, oscillation, or bifurcation which may be harmful to a system [14]. From the point of view of engineering applications, periodic

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oscillation in neural networks is an interesting dynamic behavior, as many biological and cognitive activities require repetition (e.g., heartbeat, respiration, mastication, locomotion, and memorization). In addition, periodic oscillations in neural networks have found many applications, such as associative memories, pattern recognition, machine learning, and robot motion control, see [15]. Thus, the study of periodic neurodynamics with consideration of the delayed problem becomes extremely significant to manufacture high quality neural networks.

On the other hand, as pointed out by Gotarredona et al. in [16,17], second-order cellular neural networks with reaction–diffusion terms have been identified which are able to reproduce through parameter setting a rich variety of spatio-temporal behaviors, which can be capable to robustly reproduce the rich phenomenology associated with active wave propagation and pattern formation. These wave formation phenomena are exhibited by systems belonging to very different scientific disciplines, for example, in neurophysiology, the propagation of electrical impulses through the nervous system, or the propagation of the cardiac movement through the cardiac muscle. The reaction–diffusion effects, therefore, cannot be ignored in both manmade and biological neural networks, especially when electrons are moving in a noneven electromagnetic field. So we must consider that the neuron activations vary in time as well as in space, and in this case the model should be expressed by partial differential equations. Thus, there has been an increasing interest in the study of qualitative analysis such as stability, periodicity or synchronization of neural networks with reaction–diffusion terms and delays and a large number of works have been reported in the literature, see [18–27] and the references therein. However, it is worth noting that all the connection weight coefficients and external input in the references mentioned above are constants, or the external input is periodic, and to the best of our knowledge, few published papers consider the periodic attractor issue of reaction–diffusion high-order Hopfield neural networks with time-varying delays, and all the results in the references mentioned above cannot be directly extended to study such a problem.

Motivated by the above discussions, the aim of this paper is to study the existence and global exponential stability of periodic mild solution for high-order Hopfield reaction–diffusion neural networks with time-varying delays subject to Dirichlet boundary conditions, as well as positive effects of diffusion terms on existence and exponential stability of periodic mild solution. Based on the Lyapunov stability theory, we establish some novel and easily verified exponential stability criteria which also guarantee the network will be exponentially convergent to the periodic solution. The theoretical methods developed in this paper have universal significance and can be easily extended to study many other types of reaction–diffusion neural networks with delays.

The rest of this paper is organized as follows. In Section 2, the considered model of high-order Hopfield neural networks with reaction–diffusion terms and time-varying delays is presented. Some preliminaries are also given in this section. In Section 3, the existence and global exponential stability of periodic mild solutions for the considered model are studied. Then, in Section 4, a numerical example is presented to show the correctness of the theoretical analysis. Finally, Section 5 concludes this paper.

Notation. Set $H^1 = \left\{u | u, \frac{\partial u}{\partial x_k} \in L^2(\Omega), k = 1, 2, \ldots, m\right\}$, $L^2(\Omega)$ is the space of real functions on Ω which are L^2 for the Lebesgue measure, C_c^∞ is the space of infinitely differentiable functions with compact support in Ω, and $H_0^1(\Omega)$ is closure of C_c^∞ in Sobolev space H^1.

$(L^2(\Omega))^n$ is the space of real functions $(u_1, u_2, \ldots, u_n)$ on Ω, where $u_i (i = 1, 2, \ldots, n)$ is L^2 for the Lebesgue measure. It is a Banach space equipped with the norm $\|u\|_{L^2} = \max_{i=1,2,\ldots,n} \|u_i\|_{L^2}$, where $\|u_i\|_{L^2} = \left(\int_\Omega |u_i(x)|^2 dx\right)^{\frac{1}{2}}$. If no confusion arises, $\|\cdot\|_{L^2}$ is simply denoted by $\|\cdot\|$.

Let $C(\mathbb{R}, (L^2(\Omega))^n)$ be the space of functions which are continuous in t and L^2 in x.

Denote

$$\underline{c}_i = \inf_{t\in\mathbb{R}} c_i(t), \qquad \bar{c}_i = \sup_{t\in\mathbb{R}} c_i(t), \qquad \bar{a}_{ij} = \sup_{t\in\mathbb{R}} |a_{ij}(t)|, \qquad \bar{b}_{ijl} = \sup_{t\in\mathbb{R}} |b_{ijl}(t)|,$$
$$\bar{I}_i = \sup_{t\in\mathbb{R}} |I_i(t)|, \qquad \bar{I} = \max_{1\le i\le n} \bar{I}_i,$$
$$\underline{D}_i = \min_{1\le k\le m} D_{ik}, \quad \text{and} \quad \kappa = \left\{\max_{1\le i,j\le n} \sup_{t\in\mathbb{R}} \tau_{ij}(t), \max_{1\le i,k,l\le n} \sup_{t\in\mathbb{R}} \sigma_{ikl}(t), \max_{1\le i,k,l\le n} \sup_{t\in\mathbb{R}} \delta_{ikl}(t)\right\}.$$

2. Preliminaries

From the point of view of engineering applications, the effects of reaction–diffusion in biological and manmade neural networks are inevitable especially when electrons are moving in a noneven electromagnetic field [18]. A single high-order Hopfield neural network with time-varying delays and reaction–diffusion terms can be described by

$$\begin{aligned}\frac{\partial u_i(t,x)}{\partial t} &= \sum_{k=1}^{m} D_{ik}\frac{\partial^2 u_i(t,x)}{\partial x_k^2} - c_i(t)u_i(t,x) + \sum_{j=1}^{n} a_{ij}(t)f_j(u_j(t-\tau_{ij}(t),x)) \\ &\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)f_j(u_j(t-\sigma_{ijl}(t),x))f_l(u_l(t-\delta_{ijl}(t),x)) + I_i(t), \end{aligned} \tag{2.1}$$

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