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Rational solutions and lump solutions to the generalized (3 + 1)-dimensional Shallow Water-like equation☆

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ABSTRACT

Through symbolic computation with Maple, the rational solutions and the lump solutions of the generalized (3 + 1)-dimensional Shallow Water-like equation are presented by using the generalized bilinear operator when the parameter $p = 3$. It is pointed that these rational solutions are classified into five classes because they are obtained mainly by depending on the polynomial solutions. The resulting lump solutions which are rationally localized in all directions in the space are acquired by making use of the quadratic function. However, not every nonlinear partial differential equation has the lump solutions, if any, the quantity of the lump solutions is fewer than the rational solutions. Only one class of lump solutions of the generalized (3 + 1)-dimensional Shallow Water-like equation is gotten and three 3D plots with specific values of the involved parameters are plotted.

1. Introduction

Lots of natural phenomena in the physics and in science can be described by nonlinear evolution equations [1–3]. Therefore, the study of solutions of the nonlinear evolution equations becomes the main problem we concerned, such as, rational solution [4–7], algebro-geometric solution [8] and so on. In recent years, rational solutions attracted a lot of attention. Among these rational solutions, rogue wave solution [9,10] is localized in both space and time which is first observed in the deep ocean. Rogue wave's wave height could reach up to 20–30 m and appear from nowhere and disappear without trace, furthermore, its deterrent force is very fortissimo and can result in unexpected disaster in the world [11,12]. Because of its catastrophic influence, this devastating phenomenon aroused the research interest of the professors. Due to this unique natural character, it become a hot topic in many fields, for example, in the oceanographic engineering [13], capillary flow [14], atmosphere [15], finance [16] and so on [17]. Professors studied the integrable evolution equations, such as, the nonlinear schrödinger equation [18,19], the derivative NLS equation [20]. Later on, they also discussed the coupled systems, for instance the coupled NLS system which sublimed the uncoupled system and presented the interaction of the wave to wave. While the other rational solution–lump solution [21–26] also has been brought into focus recently. Contrast with the rogue solution, the lump solution is a kind of rational solution which is localized in all directions in the space, however, as to some special nonlinear equation, both of these two rational solutions will be generated with the aid of the polynomial solutions. There are only few equations that can be got the lump solutions. Such as, the three-dimensional three wave resonant interaction equation [26], p-gKP and p-gBKP equations [24], the Hirota bilinear equation [23], Kadomtsev–Petviashvili equation [25].

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In this paper, based on the generalized $(3+1)$-dimensional Shallow Water equation whose Grammian solutions have been acquired [27], we first introduce the generalized $(3+1)$-dimensional Shallow Water-like equation with the help of the generalized bilinear operator [28–32]. Then compute the rational solutions of the generalized $(3+1)$-dimensional Shallow Water-like equation by virtue of the polynomial function and classify rational solutions into five classes. In the end, we obtain the lump solutions of the generalized $(3+1)$-dimensional Shallow Water-like equation through the positive quadratic function solutions.

2. The generalized bilinear operator

The generalized $(3+1)$-dimensional Shallow Water equation is

$$u_{xxxy}+3u_{xx}u_y+3u_xu_{xy}-u_{yt}-u_{xz}=0. \tag{1}$$

With the Cole–Hopf transformation $u=2(\ln f)_x$, the bilinear form is

$$(D_x^3D_y-D_yD_t-D_xD_z)f\cdot f=0, \tag{2}$$

where the operator D is original Hirota bilinear operator [33].

In this section, we introduce the Shallow Water-like equation inspired by the classics bilinear form as follows:

$$(D_{p,x}^3D_{p,y}-D_{p,y}D_{p,t}-D_{p,x}D_{p,z})f\cdot f=0, \tag{3}$$

where the D_p operator is defined by [30–32]

$$(D_{p,x_1}^{n_1}\cdots D_{p,x_m}^{n_m}f\cdot g)(x_1,\ldots,x_m)=\prod_{i=1}^{m}\left(\frac{\partial}{\partial x_i}+\alpha_p\frac{\partial}{\partial x_i'})^{n_i}f(x_1,\ldots,x_m)g(x_1',\ldots,x_m')\right)\Big|_{x_1'=x_1,\ldots,x_m'=x_m}$$

$$=\prod_{i=1}^{m}\sum_{l_i=0}^{n_i}\alpha_p^{l_i}\binom{n_i}{l_i}\frac{\partial^{n_i-l_i}}{\partial x_i^{n_i-l_i}}f(x_1,\ldots,x_m)\times\frac{\partial^{l_i}}{\partial x_i^{l_i}}g(x_1,\ldots,x_m), \tag{4}$$

where $n_1,n_2,\ldots,n_m$ are arbitrary nonnegative integers, and the computation of the mth power of α is as follows:

$$\alpha_p^m=(-1)^r(m),\quad \text{where } m\equiv r(m)\bmod p.$$

Consider the definition of the generalized bilinear operator D_p, we let $p=3$, then Eq. (3) is equivalent to the following formula,

$$(D_{3,x}^3D_{3,y}-D_{3,y}D_{3,t}-D_{3,x}D_{3,z})f\cdot f=6f_{xx}f_{xy}+2f_yf_t-2f_{yt}f+2f_xf_z-2f_{xz}f. \tag{5}$$

Based on the generalized bilinear form, we can show the generalized $(3+1)$-dimensional Shallow Water-like equation under the transformation $u=2(\ln f)_x$ as:

$$\frac{(D_{3,x}^3D_{3,y}-D_{3,y}D_{3,t}-D_{3,x}D_{3,z})f\cdot f}{f^2}=\frac{3}{2}u_xu_y+\frac{3}{4}uu_x+\frac{3}{4}u^2u_y\partial_x^{-1}u_y+\frac{3}{8}u^3\partial_x^{-1}u_y-\partial_x^{-1}u_{yt}-u_z=0. \tag{6}$$

3. The rational solutions to the $(3+1)$-dimensional Shallow Water-like equation

In this section, we want to discuss the rational solutions to the generalized Shallow Water-like equation by using the polynomial solutions.

Let

$$f=\sum_{i=0}^{3}\sum_{s=0}^{3}\sum_{l=0}^{2}\sum_{j=0}^{2}c_{islj}x^iy^sz^lt^j, \tag{7}$$

where the c_{islj}'s are constants, we can acquire 6 classes of polynomial solutions to Eq. (5). Among the 6 classes of solutions, we enumerate 5 classes of solutions as follows:

$$f=\frac{1}{27{c_{3,1,1,2}}^4}(t^2c_{3,1,1,2}z+t^2c_{3,1,0,2}+(zc_{3,1,1,1}+c_{3,1,0,1})t+c_{3,1,1,0}z+c_{3,1,0,0})((9x^2y-243z){c_{3,1,1,2}}^3$$
$$+(9x^2c_{3,0,1,2}+6xyc_{2,1,1,2}+9c_{1,0,1,2}+243c_{3,1,0,2}){c_{3,1,1,2}}^2+c_{2,1,1,2}(6xc_{3,0,1,2}+yc_{2,1,1,2})c_{3,1,1,2}$$
$$-2{c_{2,1,1,2}}^2c_{3,0,1,2})(3xc_{3,1,1,2}+c_{2,1,1,2}), \tag{8}$$

$$f=\frac{1}{{c_{2,3,1,2}}^2}(t^2zc_{2,3,1,2}+c_{2,3,0,2}t^2+(zc_{2,3,1,1}+c_{2,3,0,1})t+c_{2,3,1,0}z+c_{2,3,0,0})(y^2(x^2y-18z){c_{2,3,1,2}}^2$$