Computers and Mathematics with Applications 73 (2017) 71–86

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

# Efficient mesh deformation based on Cartesian background mesh

Hong Fang [a], Chunye Gong [b,c,d,*], Caihui Yu [a], Changwan Min [a], Xing Zhang [a], Jie Liu [b], Liquan Xiao [b]

[a] *Science and Technology on Space Physics Laboratory, Beijing 100076, China*
[b] *Computer School, National University of Defense Technology, Changsha 410073, China*
[c] *College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China*
[d] *State key laboratory of Space Weather, Chinese Academy of Science, Beijing 100190, China*

ARTICLE INFO

*Article history:*
Received 15 December 2015
Received in revised form 28 October 2016
Accepted 31 October 2016
Available online 13 December 2016

*Keywords:*
Mesh deformation
Cartesian background mesh
Radial basis functions
Greedy algorithm

ABSTRACT

Moving mesh is widely used in the simulation of aerodynamic shape optimization, multibody relative motion, aircraft icing and aeroelasticity. The efficient and high quality mesh deformation is the key technology of moving mesh. This paper presented a new Mesh Deformation method based on Cartesian Background Mesh (MDCBM). First, the Cartesian background mesh is deformed with radial basis functions (RBF). Second, the displacement of Cartesian background mesh is algebraically interpolated onto all meshes in the computing domain. Since the background mesh is coarse, the background mesh deformation can be finished fast. Because the background mesh of MDCBM is regular, the mapping relationship between background mesh and the computing mesh is simple. So the time spent on mapping search is substantially reduced. The examples including NACA0012 airfoil, multi-element airfoil with structured, unstructured mesh and DLR-F4 wing–body show the good performance of MDCBM. We highlight the advantages of MDCBM with respect to its computational efficiency and high quality of deformed mesh.

## 1. Introduction

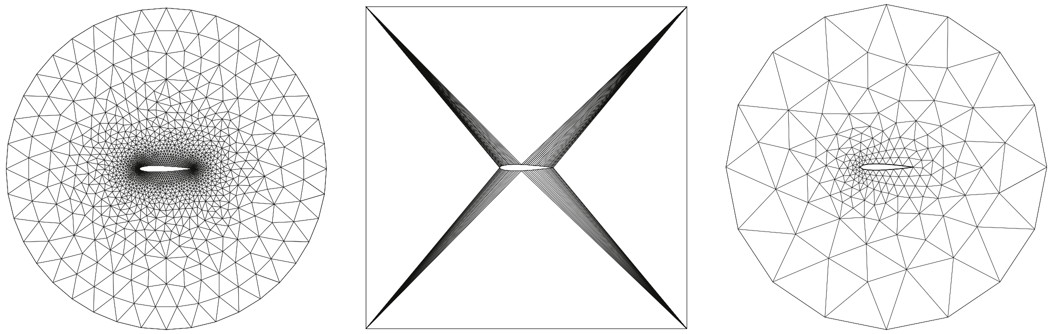
Moving mesh is widely used in the simulation of optimum aerodynamic design, multibody relative motion, aircraft icing and aeroelasticity et al. [1,2]. The efficient and high quality mesh deformation is the key technology for moving mesh [3]. At present, the spring analogy method [4], linear elasticity analogy method [5], RBF-based method [6], background mesh based methods [7,8], PDE-based methods [9], Finite Macro-Element Mesh Deformation methods [10], optimization-based methods [11] and other methods [12] are developed for mesh deformation. The spring analogy method was first presented by Batina [4]. The basic idea of spring analogy method is to create a network of springs connecting all nodes in the mesh and the stiffness of springs is inversely proportional to the edge length. The displacement of each node is got by solving a balancing spring system. If the local mesh motion is much bigger than the local mesh size, the classical spring analogy method will fail [13].

The spring analogy method is widely applied in aeroelastic simulation and aerodynamic shape optimization, but lacks robustness in deforming refined boundary layer viscous meshes for large deformations. The robustness in deforming viscous

---

[*] Corresponding author at: College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China.
*E-mail address:* gongchunye@nudt.edu.cn (C. Gong).

http://dx.doi.org/10.1016/j.camwa.2016.10.023



(a) Original mesh (3156 Cells).

(b) Delaunay graph mapping background mesh (84 Cells).

(c) Unstructured background mesh (332 Cells).

**Fig. 1.** Original mesh and background mesh.

meshes with large deformations was improved by Farhat et al. [14] by using a torsional spring to avoid the grid crossover problem. The torsional spring is shown to be essential for moving viscous meshes [15]. The linear elasticity analogy method, which can be viewed as a good example of PDE-based method, models deformation domain as an elastic solid [5,16,17]. This method is very robust in large deformation problems and results in high quality mesh. However, the computational cost of this method for large meshes is much higher than that of the spring analogy due to the computational complexity of solving finite element equations. Finite macro-element mesh deformation method is an extension of the linear elasticity analogy [10]. It consists of two steps. The first step is a finite element solution of macro-elements created from a subset of points from all meshes. The second step is a transfinite interpolation which distributes the macro-element nodal deflections to all meshes. The computing cost for solving the finite element system on macro-elements is substantially lower than on all elements and the quality of deformed mesh is much higher. But it is only suitable for structured mesh. The CFL3D developed by NASA Langley Research Center adopts this mesh deformation method. Other PDE-based methods need to solve Laplacian equations [1], weighted Laplacian smoothing [18], biharmonic partial differential equations [19] with Dirichlet boundary conditions. Optimization-based methods, such as target-matrix paradigm [11,20], focus on mesh quality of deformation.

RBF method is a very effective data interpolation algorithm for multivariate scattered nodes [21–23]. The basic principle of RBF-based mesh deformation technology is to set up RBF sequences by interpolating the displacement of boundary nodes, and then the displacement of boundary nodes are scattered to the whole computing nodes by the RBF sequences smoothly. And then the displacement of boundary nodes is scattered to the whole computing nodes by making use of RBF sequences smoothly. This method can effectively deal with mesh deformation with large boundary displacement. This method has several significant advantages. Primarily, it can effectively deal with mesh deformation with large boundary displacement and complex deforming mode. Then, it may be convenient for parallelism and equally well be applied to structured mesh, unstructured mesh and point clouds because no connectivity information is required [24]. With the development and improvement of greedy algorithm [25], multi-level subspace algorithm [26] and 'double-edged' greedy algorithm [27], the efficiency of this method is improved vastly due to substantial reduction of RBF support nodes. The RBF method is widely applied to different areas, such as aeroelastic [28], aircraft icing [27], fluid–structure interaction interface interpolation [29] and the numerical solutions of partial differential equation. In general, the RBF support nodes after reduced with greedy algorithms are about 300–1000 support nodes in the real engineering applications. If the displacement of each node is calculated by RBF interpolation, the amount of computation will be tremendous.

The basic idea of background mesh method is to construct background mesh from original mesh, deform the background mesh by universal mesh deformation algorithm, and then interpolate the displacement of background mesh back to the original mesh. Background mesh method has two advantages. First, because the background mesh is coarse, the computational efficiency is high. Second, the background mesh method is more robust for large deformation. Delaunay graph mapping method (DGMM) [8] is one kind of background mesh methods. Delaunay graph here means Delaunay triangulation, which maximizes the minimum angle of a triangle or tetrahedral. The Delaunay graph provides a unique mapping between the given boundary points and a coarse unstructured mesh. DGMM generates a Delaunay graph from all computational nodes of the solution domain. Maintaining the mapping relationship during the movement, the new computational mesh can be obtained efficiently by relocating the nodes in the graph. The original mesh with 3156 cells is shown in Fig. 1(a). The DGMM background mesh with 84 cells is shown in Fig. 1(b). Because DGMM does not need to solve linear systems, it is non-iterative and much more efficient. Computing deformation of any node only needs the interpolations of three or four boundary nodes. Compared with PDE-based method or analogy, DGMM is much more efficient. But for mesh with complex shape, how to effectively build Delaunay graph is still under study. Mesh deformation based on unstructured background mesh method is presented by Zhou et al. [7] and very useful for unstructured mesh deformation in engineering applications. The approach generates a background mesh (coarse mesh) of the solution domain (shown in Fig. 1(c)) at first. Then the deformation of the background mesh caused by boundary movements is performed by a spring analogy. Finally, the deformation of the computing mesh is obtained by an interpolation method with the background mesh.