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A column generation approach for the location-routing problem with time windows

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ABSTRACT

The location-routing problem with time windows consists of opening a subset of depots, assigning customers to depots, and determining routes within allowable times such that the sum of depot opening, vehicle usage, and traveling costs is minimized. Customers have to be visited only once during their time windows and depot capacities and load limits of vehicles cannot be violated. In order to find the exact solution to the problem, we propose a branch-and-price algorithm based on set-partitioning approach. The pricing problem is solved using dynamic programming. We introduce several strategies to improve the lower and upper bounds as well as acceleration techniques to generate improving columns more rapidly. Computational results show the higher performance of the proposed method on a set of small and medium size instances in the literature and demonstrate its efficiency in solving generated large size instances.

1. Introduction

Known as one of the integrated problems in logistics, the location-routing problem (LRP) consists of two basic decisions to be made, each of which known as a hard combinatorial optimization problem: decisions on the location of facilities (such as depots, warehouses, etc.) and decisions on the routing of vehicles. Although these two types of problems have been traditionally considered separately at different planning levels, it is shown that the integrated approach of the LRP reduces overall cost in the long planning horizon (Nagy and Salhi, 2007; Salhi and Nagy, 1999). Urban freight transportation, food and drink distribution, waste collection, postbox location, and blood bank location are some examples of practical LRP applications (Nagy and Salhi, 2007).

In the LRP, a set of potential capacitated facility locations, a set of customer locations with known demands, and a fleet of delivery vehicles with limited capacity are given. We assume fixed costs for opening the facilities and using vehicles. The problem is to decide on which facilities to open and which vehicle routes to use such that total facility opening cost, vehicle usage cost, and traveling cost is minimized. Total customer demands assigned to each facility should not exceed its capacity. Capacity of each vehicle also limits total customer demands to be delivered. The reader is referred to Lopes et al. (2013), Prodhon and Prins (2014) and Drexl and Schneider (2015) for recent review of the LRP literature, its taxonomy, and the problem characteristics. Here, we focus our attention to the recent exact methods developed for the LRP. Akca et al. (2009) present a set-partitioning formulation of the problem and a branch-and-price algorithm where the pricing problem is solved using dynamic programming. Belenguer et al. (2011) develop a branch-and-cut algorithm based on the mixed integer programming (MIP) formulation of the problem strengthened by valid inequalities. Contardo et al. (2013) extended the work of Belenguer et al. (2011) by introducing valid inequalities and present separation algorithms that are either new or that generalize the separation methods introduced by Belenguer et al. (2011). Baldacci et al. (2011b) propose a bounding procedure for the LRP and decompose the problem into a limited set of capacitated multi-depot vehicle routing problems. Contardo et al. (2014) introduce valid inequalities for the set-partitioning formulation of the LRP and propose a branch-and-cut-and-price algorithm to improve the results obtained by Baldacci et al. (2011b).

In this study, we consider the LRP where serving a customer has to be started during a predefined time interval specified to the customer. This problem is called the location-routing problem with time windows (LRPTW). Although time window restrictions have been considered for the vehicle routing problem (namely VRPTW) for a long time, it is not studied in the context of LRPs. Ponboon et al. (2016) provide an arc-flow MIP formulation of the LRPTW and develop a branch-and-price algorithm based on the set-partitioning formulation of the problem. The pricing problem

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Table 1
Exact solution methods for the LRP.

Problem	Reference	Solution method	Problem size
LRP	Akca et al. (2009)	Branch & price	5–40
	Belenguer et al. (2011)	Branch & cut	5–50
	Contardo et al. (2013)	Branch & cut	5–50
	Baldacci et al. (2011b)	Branch & cut & price	14–199
	Contardo et al. (2014)	Branch & cut & price	14–199
LRPTW	Ponboon et al. (2016)	Branch & price	3–40
	This study	Branch & price	5–50

is defined for each candidate depot location and is solved by the labeling algorithm. For the experimentation, the authors generate new set of instances by modifying the LRP and VRPTW benchmark instances in the literature. Table 1 summarizes the exact methods proposed for the LRP and the LRPTW, and lists the largest problem instances that could be solved by each method. The problem instance size shows the number of candidate facility (depot) locations followed by the number of customers.

We contribute to the LRP literature by presenting an exact solution approach for the LRPTW based on branch-and-price for solving larger problem sizes and reducing solution times. The difference with Ponboon et al. (2016) is that our column generation framework is enhanced by various techniques inspired from the VRPTW and the LRP literature for improving lower and upper bounds on the objective function value of the LRPTW. The performance of the algorithm is tested on a large number of instances with different characteristics. For the instances in Ponboon et al. (2016), the numerical results show that the computational efforts are significantly reduced when the enhancement techniques are applied on the proposed algorithm. The efficiency of the developed branch-and-price algorithm is also shown over a set of larger size instances with 5 candidate facilities and 50 customers. We also introduce a two-stage heuristic where the first stage fixes the depot locations under time window limits and the second stage allocates customers to the depots through routing with time windows, using the branch-and-price framework. The computational experiments indicate that the heuristic saves significant amount of time to solve the problem without sacrificing the solution quality.

The reminder of this paper is as follows. In Section 2 we provide the arc-flow MIP formulation of the LRPTW and its decomposed problems. We show how the proposed techniques are adopted to improve the lower bound and solve the problem more efficiently in the subsequent sections. Section 3 introduces the labeling algorithm used to solve the subproblems and explains how this algorithm is accelerated. Various techniques are presented for improving the lower bound in Section 4. The branching scheme used to reach the integer solution and a number of valid inequalities added to the master problem are introduced in Section 5. Section 6 is for finding good solutions in short times through using a simple two-stage heuristic. Section 7 presents the problem test instances and the experimental results of the proposed solution approach. We conclude the paper and suggest potential research directions in Section 8.

2. Problem formulation

In this section, we provide the arc-flow formulation as well as the set-partitioning formulation of the LRPTW.

2.1. Arc-flow formulation

Suppose the set of customers is given by set N. Each customer $i \in N$ is characterized by unsplittable demand d_i and time window $[a_i, b_i]$. Like most VRPTW studies, we assume that a vehicle is free to arrive at a customer location before its time window begins. However, serving customer i cannot be started earlier than a_i or later than b_i. Let M be the set of potential depot locations. Each depot $m \in M$ has opening fixed cost H_m and capacity Q_m and is available during working hours $[0, b_m]$. There is a homogeneous fleet of vehicles with fixed usage cost h and capacity q given as set K. It is assumed that sufficient number of vehicles can depart from any depot at time 0. The LRPTW is defined on a network created by the vertex set $V = M \cup N$ and arc set $A = (i, j), \forall i, j \in V$ excluding depot-to-depot arcs. Define time t_{ij} to include service time at location i and the time required to travel from i to j. Let B_{ij} be equal to $\max\left\{b_i + t_{ij} - a_j, 0\right\}$. Traveling on arc $(i, j) \in A$ imposes operating cost c_{ij}.

For each depot m, a binary decision variable y_m takes value 1 if the depot is opened. To determine vehicle routes, a binary decision variable x_{ij}^k is defined to take value 1 if vehicle $k \in K$ travels on arc $(i, j) \in A$. Decision variable s_i^k denotes the arrival time of vehicle k to location $i \in V$. Hence, s_m^k denotes the total service and travel time spent by vehicle k when it returns to depot $m \in M$. Given this notation, the arc-flow formulation of the LRPTW is as follows.

$$\text{minimize} \sum_{m \in M} H_m y_m + h \sum_{m \in M} \sum_{i \in N} \sum_{k \in K} x_{mi}^k + \sum_{(i,j) \in A} \sum_{k \in K} c_{ij} x_{ij}^k \tag{1}$$

$$\text{subject to} \sum_{k \in K} \sum_{i \in V} x_{ij}^k = 1 \;\; \forall j \in N \tag{2}$$

$$\sum_{m \in M} \sum_{i \in N} x_{mi}^k \le 1 \;\; \forall k \in K \tag{3}$$

$$\sum_{i \in V} x_{il}^k - \sum_{j \in V} x_{lj}^k = 0 \;\; \forall k \in K, l \in N \tag{4}$$

$$\sum_{i \in N} \sum_{j \in V} d_i x_{ij}^k \le q \;\; \forall k \in K \tag{5}$$

$$\sum_{k \in K} \sum_{i \in N} \sum_{j \in N \cup \{m\}} d_i x_{ij}^k \le Q_m y_m \;\; \forall m \in M \tag{6}$$

$$t_{mi} - s_i^k \le B_{mi}\left(1 - x_{mi}^k\right) \;\; \forall k \in K, m \in M, i \in N \tag{7}$$

$$s_i^k + t_{ij} - s_j^k \le B_{ij}\left(1 - x_{ij}^k\right) \;\; \forall k \in K, i \in N, j \in V \tag{8}$$

$$a_i \le s_i^k \le b_i \;\; \forall k \in K, i \in V \tag{9}$$

$$y_m \in \{0, 1\} \;\; \forall m \in M \tag{10}$$

$$x_{ij}^k \in \{0, 1\} \;\; \forall k \in K, (i, j) \in A \tag{11}$$

The objective function (1) minimizes total depot opening cost, vehicle fixed cost, and traveling cost. Constraint (2) ensures that each customer is visited exactly once. By constraint (3) each vehicle is used at most once. Flow balance at each location is preserved by constraint (4). Capacity of vehicles and depots are controlled by constraints (5) and (6), respectively. Constraints (7) and (8) make the vehicle traveling on arc (i, j) visit i before j. Constraint (9) limits arrival time to each location to its time windows. Binary requirements are met by constraints (10) and (11).

The formulation (1)–(11) contains a large number of variables and constraints even for small-size instances. The number of binary variables is in the order of $|V|^2|K|$ and the number of constraints is in the order of $|N|^2|K|$. Considering only 3 candidate