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A capacitated hub location problem under hose demand uncertainty

Merve Meraklı[a,b,*], Hande Yaman[a]

[a] Department of Industrial Engineering, Bilkent University, Ankara, Turkey
[b] Department of Integrated Systems Engineering, The Ohio State University, Columbus, OH, USA

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ABSTRACT

In this study, we consider a capacitated multiple allocation hub location problem with hose demand uncertainty. Since the routing cost is a function of demand and capacity constraints are imposed on hubs, demand uncertainty has an impact on both the total cost and the feasibility of the solutions. We present a mathematical formulation of the problem and devise two different Benders decomposition algorithms. We develop an algorithm to solve the dual subproblem using complementary slackness. In our computational experiments, we test the efficiency of our approaches and we analyze the effects of uncertainty. The results show that we obtain robust solutions with significant cost savings by incorporating uncertainty into our problem.

1. Introduction

Hubs are used commonly in many-to-many distribution systems that arise in transportation and telecommunications applications. Flows from many origins to many destinations are consolidated at hubs and routed together to benefit from economies of scale. Many variants of hub location problems have been studied in the last few decades. Given a set of nodes with pairwise traffic demands, the hub location problem decides on the locations of the hubs and the routes of traffic demands to minimize some performance measure. This measure can be related with the system cost or the quality of service. The system cost includes the cost of routing the traffic in the hub network and it may include the fixed cost of locating hubs if the number of hubs is not fixed. In some variants, direct shipments between nonhub nodes are allowed, in others all the traffic is routed through at least one hub. Also, there are variants of the problem where a nonhub node can send and receive traffic through multiple hubs and others where there is a restriction on the number of hubs that a nonhub node can use. The first setting is known as the multiple allocation setting. In this paper, we study a hub location problem with multiple allocation, fixed costs for installing capacitated hubs and no direct shipments.

Most studies in the hub location literature are based on the assumption that the pairwise demands are known with certainty. However, this is very difficult to justify in practice since strategic decisions such as hub location decisions are often taken before observing the actual demand and the demand fluctuates over time. In this study, we incorporate the demand uncertainty into the capacitated multiple allocation hub location problem. In this setting, demand uncertainty affects both the feasibility of a hub network and its associated cost. To hedge against demand uncertainty, we use a robust optimization framework: among all hub networks that are feasible for all possible demand realizations, we would like to find one that minimizes the worst case total cost (for more on robust optimization see, e.g., Atamtürk (2006); Ben-Tal et al. (2004); Ben-Tal and Nemirovski (1998); 1999); 2008); Bertsimas and Sim (2003); 2004); Mudchanatongsuk et al. (2008); Ordóñez and Zhao (2007); Yaman et al. (2001); 2007)).

We represent the uncertainty with a special polyhedral uncertainty model known as the hose model. The parameters of this model are aggregate traffic upper bounds for each node. Any nonnegative demand vector in which the sum of traffic demands that each node can send and receive does not exceed the traffic upper bound for that node is a possible demand realization. The hose model was proposed by Duffield et al. (1999) and Fingerhut et al. (1997) to design virtual private networks. It has several advantages compared to other uncertainty models: it asks to estimate a parameter for each node rather than for each pair of nodes. This aggregation reduces the statistical variability and errors. It has resource-sharing flexibility and is not a conservative

* Corresponding author at: Department of Integrated Systems Engineering, The Ohio State University, Columbus, OH, USA.
E-mail addresses: merve.merakli@bilkent.edu.tr (M. Meraklı), hyaman@bilkent.edu.tr (H. Yaman).

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model in which each origin-destination traffic demand can take its worst case value simultaneously. Due to these advantages, the hose model has been used as an uncertainty model in many studies following its introduction (some examples are Altın et al. (2007); 2011); Chekuri et al. (2007); Italiano et al. (2006)).

Recently, Meraklı and Yaman (2016) study the uncapacitated multiple allocation p-hub median problem with polyhedral demand uncertainty. They present a mixed integer programming model and apply Benders decomposition. Their results show that algorithms based on decomposition are very efficient compared to solving the model with an off-the-shelf solver. They also observe that it is possible to obtain significant cost savings by incorporating demand uncertainty into the problem. In the uncapacitated problem, the demand only affects the routing costs. In addition, it is known that when hub locations are given, each traffic demand is routed on a shortest path from its origin to its destination independently of the amount of demand. As a result, it is possible to hedge against uncertainty with minor changes in the network. These are not true when capacity constraints are imposed for hubs.

In this paper, we present a model for the capacitated hub location problem with multiple allocation and hose demand uncertainty. Our initial computational experiments showed that the model is much harder to solve compared to its deterministic counterpart. We propose two exact algorithms based on Benders reformulations and give an algorithm to solve the dual subproblem using complementary slackness. We test the efficiency of these algorithms using instances from the literature. We also perform experiments to investigate the changes in the hub locations and costs as a result of demand uncertainty. We observe that ignoring demand uncertainty may result in high routing costs and congested hubs. Unlike the observations for the uncapacitated problem, when capacity constraints are imposed, one may need to make major changes in the hub locations to hedge against uncertainty.

The rest of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we first present a nonlinear model and then derive a compact linear mixed integer programming model. We give two Benders reformulations in Section 4. We report the results of computational experiments in Section 5 and conclude the paper in Section 6.

2. Literature review

In the last few decades, hub location problems have received a lot of attention both in telecommunications and transportation literatures. Here we limit ourselves to related studies and refer the reader to surveys in Campbell (1994b), and Alumur and Kara (2008); Campbell et al. (2002); Campbell and O'Kelly (2012); Klincewicz (1998); O'Kelly and Miller (1994) and Farahani et al. (2013) for further information.

The multiple allocation hub location problem is first formulated by Campbell (1994a). Boland et al. (2004); Camargo et al. (2008); Cánovas et al. (2007); Ebery et al. (2000); Ernst and Krishnamoorthy (1998a); Hamacher et al. (2004); Klincewicz (1996); Marín (2005b); Mayer and Wagner (2002) and Contreras et al. (2011a) propose methods to solve this problem. The version of the problem where there is no cost for opening hubs but the number of hubs is fixed to p is first formulated by Campbell (1992). Alternative formulations are given by Campbell (1994a); Skorin-Kapov et al. (1996) and Ernst and Krishnamoorthy (1998a). Campbell (1996) and Ernst and Krishnamoorthy (1998a); 1998b) propose exact and heuristic solution algorithms.

Among the studies cited above, several propose Benders decomposition based approaches. Camargo et al. (2008) propose three different algorithms: the classical Benders decomposition approach, which adds a single cut at each iteration, a multi-cut version in which Benders cuts are generated for each origin-destination pair and a variant which terminates when an ϵ-optimal solution is obtained. Contreras et al. (2011a) propose a Benders decomposition in which they generate cuts for each candidate hub location instead of each origin-destination pair. Camargo et al. (2009) propose two Benders decomposition algorithms to solve the variant of the problem where the cost is a piecewise-linear concave function. Gelareh and Nickel (2011) study a problem with an incomplete hub network and solve this problem with a Benders decomposition algorithm.

Capacitated variants of the hub location problems received less attention in the literature compared to the uncapacitated versions. The first mixed integer linear programming formulation for the capacitated multiple allocation hub location problem (CMAHLP) is proposed by Campbell (1992) using four indexed variables. Ebery et al. (2000) provide formulations with three indices and devise a heuristic algorithm to solve large instances. In order to strengthen these formulations, Boland et al. (2004) propose preprocessing procedures and valid inequalities, which lead to a significant reduction in the computation times. Marín (2005a) also provides new formulations and resolution techniques to obtain better computational results and succeeds to solve instances with up to 75 nodes. Sasaki and Fukushima (2003) consider a capacitated multiple allocation hub location problem where a capacity constraint is applied both on hubs and arcs and a flow can go through at most one hub on its way from origin to destination. They devise a branch and bound algorithm and perform computational studies on the CAB data set.

There are also Benders decomposition applications for the capacitated multiple allocation hub location problems. Rodríguez-Martín and Salazar-González (2008) consider a capacitated hub location problem with multiple allocation on an incomplete hub network. They provide a formulation and develop two exact solution algorithms. The first one utilizes classical Benders decomposition approach whereas the second employs a nested two level algorithm based on Benders decomposition. They show that the latter outperforms the classical Benders decomposition approach in terms of computation times. Contreras et al. (2012) also study a related capacitated hub location problem in which the capacities installed on each hub is not a parameter but a decision variable. They devise a Benders decomposition algorithm in which the subproblem is a transportation problem. They apply Pareto-optimal Benders cuts and reduction tests to improve the convergence of the algorithm.

The studies that incorporate data uncertainty into hub location problems is rather limited. Marianov and Serra (2003) study the problem in an air transportation network where hubs are $M/D/c$ queues and the probability that the number of planes in the queue exceeds a certain number is bounded above. This restriction is then reformulated as a capacity constraint for the hubs. The authors propose a tabu search based heuristic method to solve this problem. Yang (2009) decides on hub locations and flight routes under demand uncertainty using two-stage stochastic programming. The first stage involves the decision on the locations of the hubs to open. In the second stage, routes are determined after demand realizations are observed. Sim et al. (2009) incorporate service level considerations using chance constraints when travel times are normally distributed. They propose several heuristic algorithms. Contreras et al. (2011b) consider the uncapacitated multiple allocation hub location problem under demand and transportation cost uncertainty. They show that the stochastic models for this problem with uncertain demands or transportation costs dependent to a single uncertain parameter are equivalent to the deterministic problem with mean values. This is not the case for the problem with stochastic independent transportation costs. This latter problem is solved using Benders decomposition and a sample average scheme. They use the AP data set to test the