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Empirical analysis for the VRPTW with a multigraph representation for the road network

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A B S T R A C T

Vehicle routing problems have drawn researchers' attention for more than fifty years. Most approaches found in the literature are based on the key assumption that for each pair of points of interest (e.g., customers, depot…), the best origin-destination path can be computed. Thus, the problem can be addressed via a simple graph representation, where nodes represent points of interest and arcs represent the best paths. Yet, in practice, it is common that several attributes are defined on road segments. Consequently, alternative paths presenting different trade-offs exist between points of interest. In this study, we investigate in depth a special representation of the road network proposed in the literature and called a multigraph. This representation enables one to maintain all these alternative paths in the solution space. We present an empirical analyses based on the Vehicle Routing Problem with Time Windows, as a test bed problem, solved with branch-and-price algorithms developed for the different types of graphs. Computational experiments on modified benchmarks from the literature and on instances derived from real data evaluate the impact of the modeling on solution quality.

1. Introduction

The vehicle routing problem is one of the most extensively studied classes of combinatorial optimization problems in the operational research literature. One reason is the large number and the interest of its applications in logistics, supply chain management, distribution systems, car navigation systems, etc. Although this research area has been broadly explored in the last fifty years, most works are built on an assumption that is at best disputable, and at worst can lead to a bad optimization of vehicle routes.

A vehicle routing problem aims at planning a set of routes on a given road network, so as to cover a set of customer requests with a fleet of vehicles. Most models proposed in the literature represent the road network with a weighted simple graph. Nodes are introduced for the different points of interest (e.g., customers, depot…), and arcs correspond to shortest paths computed according to a single criterion, generally traveling cost, distance or traveling time, between these points of interest.

In real-life applications and especially in urban areas, several operational constraints are implied and objectives of the involved partners must be taken into account. In many cases, different attributes have to be defined for each road segment in the original road network. Hence, each pair of nodes may be connected with a set of paths proposing different compromises between the considered attributes. In such situations, representing the problem with a simple graph, i.e., with only one arc between each pair of nodes, could discard many potentially good solutions from the solution space. To handle this issue, an alternative modeling approach, so-called multigraph representation, was proposed by Garaix et al. (2010). This representation aims at considering all non-dominated paths linking each two points of interest in the solution space.

A typical example is provided by the Vehicle Routing Problem with Time Windows (VRPTW). In this problem, transportation plans are constrained to satisfy customer requests within their time windows. Each road segment is defined with a cost and a travelling time. In the standard setting, the problem is defined on a simple graph and each arc represents a best path. However, the cheapest path is unlikely to be the same as the fastest path due to, for example, heavy traffic and congestion in some short road segments or additional charges for high-speed routes. When defining its transportation plan, a carrier might prefer an expensive road segment in case of hard time constraints or, conversely, a cheapest one when time constraints are soft. Consequently, representing the problem with a weighted simple graph could lead to operational solutions with an overestimated cost, or, even worse, to the false

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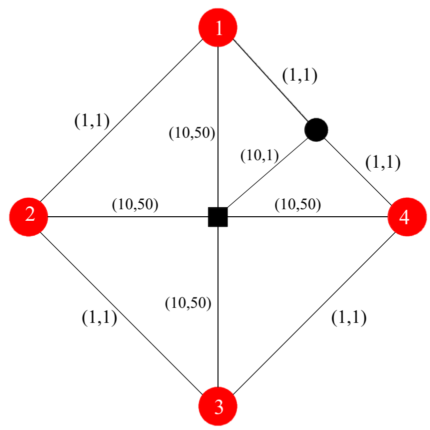

Fig. 1. Illustrative road network.

conclusion that no feasible solution exists. To illustrate this, let us consider the small road network provided on Fig. 1.

In this example, the depot is located at node 0 (represented by the black square), and customers are located at nodes 1, 2, 3 and 4. Each edge represents a road segment. The black circle node corresponds to a junction of three road segments. For each edge, the traveling cost and time are provided in parentheses, in this order *(cost, time)*. The objective is to determine a set of vehicles routes that visit all customers and return to the depot within 100 units of time.

If we consider the simple graph where arcs represent cheapest paths (Fig. 2a), the best solution costs 80 and consists in visiting each customer with a different vehicle. Considering all alternative paths, a multigraph representation (Fig. 2b) allows one to obtain a better solution that serves all customers with the same vehicle with a total traveling cost equal to 24. Note furthermore that for a lower time-limit (or for a limited fleet), no feasible solution could have been found with the simple graph representation. Note also that equivalent examples could be constructed with a simple graph obtained from fastest paths, except that in this case a feasible solution would always be found when it exists.

Following the above remarks, we investigate in this work two important issues:

1. Is it tractable to represent the road network with a multigraph, where, for each pair of nodes, the set of alternative arcs is the set of non-dominated Pareto optimal paths computed according to road segment attributes?
2. Could this representation have a significant impact on solution costs in practice?

The first issue relates to the computation and size of the multigraph, and to the efficiency of solution methods. The second issue relates to experimental analyses and comparisons between simple graph and multigraph representations. For our investigations, we use the VRPTW as a test-bed problem.

Besides the case of the VRPTW, where cost and time are associated with arcs, many other attributes might be considered in vehicle routing depending on the context. Greenhouse gas emissions (that depend on travel distances but also average speeds, slopes...) would be considered by a decision-maker who attempts to limit environmental impacts when defining its transportation plans. Energy consumption (that also depends on distances, speeds, slopes...) has to be taken into account when electric vehicles are operated. An attribute modeling scenic beauty could be introduced when optimizing sightseeing tours. Safety (for hazmat or cash transportation), transport mode, robustness define other examples.

The remainder of the paper is organized as follows. In Section 2, we review the relevant literature and give more insights into the methodology followed in this work. In Section 3, we present a mathematical formulation for the VRPTW with the multigraph representation. We describe, in Section 4, a branch-and-price solution approach adapted to the multigraph setting. Finally, in Section 5 we present the results of extensive experiments conducted to evaluate the impact of the modeling approach and the efficiency of our method.

2. Literature review

Vehicle Routing Problems are widely studied and a large number of solution approaches are proposed in the literature. The large majority of these approaches are based on the key assumption that one can compute the best path for all pairs of nodes. Thus, the problem can be tackled using a simple graph. As mentioned before, this assumption is not guaranteed to hold when several attributes are defined on arcs. In the literature, few works evoke this issue. In this section, we overview these works. In some of them, a multigraph representation is investigated. In others, the problem is directly solved in the original road network. These two options are surveyed in the two next subsections, respectively. A subsequent subsection details how our work intends to complete this literature and motivates the methodology that we proposed to follow.

2.1. Multigraph representation

The first possibility consists in representing the road network with a multigraph, so that alternative routes are considered between each pair of points of interest.

A key paper in this regard was proposed by Garaix et al. (2010). As far as we know, they were the first to point out that when several attributes are defined on arcs, one cannot transform a vehicle routing problem on a road network into a standard VRP without taking the risk of losing optimality, and to explore this issue. Their motivation stemmed from the development of a real-world *On-Demand Transportation* system. For this reason, they were guided by the objective of efficiently solving a specific Dial-a-Ride Problem. They introduced a multigraph representation and developed two specialized solution approaches: a simple insertion heuristic and a branch-and-price procedure. Experiments were mainly designed to evaluate practical objectives, but they also demonstrated that important improvements could be obtained using the multigraph representation compared to a fastest-path-based simple graph.

Lai et al. (2016) considered this issue for the heterogeneous VRP with limited duration. Following Garaix et al. (2010), they introduced alternative arcs between pair of vertices. They proposed a tabu search and insisted on how neighborhood exploration should be modified to consider the multigraph structure. Experiments were carried out on randomly generated instances with two alternative arcs between every vertex pair. The advantages of introducing the alternative arcs were largely investigated. The experiments confirmed, on a different problem, the observations made in Garaix et al. (2010).

Besides the two aforementioned papers that explicitly investigate the limits of the simple graph representation, several other papers also consider multigraphs, with relatively similar goals.

Wang and Lee (2014) introduced the so-called Time Dependent Alternative Vehicle Routing Problem (TDAVRP) that also involves a multigraph representation. The TDAVRP is a vehicle routing problem with time windows and travel times depending on the time of the day. Each pair of nodes is connected with two edges. The first one is called the designated edge and is assigned a time-dependent travel speed distribution. The other one represents an alternative