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Time-dependent green Weber problem

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ABSTRACT

We consider an extension of the classical Weber problem, named as the green Weber problem (GWP), in which the customers have one-sided time windows. The GWP decides on the location of the single facility in the plane and the speeds of the vehicles serving the customers from the facility within the one-sided time windows so as to minimize the total amount of carbon dioxide emitted in the whole distribution system. We also introduce time-dependent congestion which limits the vehicle speeds in different time periods and call the resulting problem as the time-dependent green Weber problem (TD-GWP). In the TD-GWP, the vehicles are allowed to wait during more congested time periods. We formulate the GWP and TD-GWP as second order cone programming problems both of which can be efficiently solved to optimality. We show that if the traffic congestion is non-increasing, then there exists an optimal solution in which the vehicles do not wait at all. Computational results are provided comparing the locations of the facility and the resulting carbon dioxide emissions of the classical Weber problem with those of the GWP and comparing the GWP with the TD-GWP in terms of carbon dioxide emissions in different traffic congestion patterns.

1. Introduction and literature review

A *single–facility location problem* (SFLP) is the problem of finding the location of a single facility which will serve a set of customers so as to minimize an objective function, usually a function of the distances between the facility and the customers. The (discrete) *1-center* problem (Agarwal et al., 1998), a discrete facility location problem, and the *Weber problem*, (Drezner, 1992; Drezner and Hamacher, 2002), a continuous facility location problem, are among the most famous SFLPs.

The Weber problem is the problem of locating a single facility in the plane with the aim of minimizing a weighted sum of the Euclidean distances between the facility and the customers, where the weights are positive constants. It is assumed that it is always possible to go directly from the facility to any customer. Let the location of the facility and the locations of the n customers be denoted by (x, y) and $(a_i, b_i), i \in \{1, 2, \dots, n\}$, respectively and $w_i > 0$ represent the weight of customer $i, i \in \{1, 2, \dots, n\}$. The Weber problem can be formulated as a nonlinear optimization problem as

$$\min_{(x,y)\in\mathbb{R}^2} \quad \sum_{i=1}^{n} w_i \|(x, y) - (a_i, b_i)\|, \tag{1}$$

where $\|(x, y) - (a_i, b_i)\|$ is the Euclidean distance between the points (x, y) and (a_i, b_i).

A standard approach for solving the Weber problem is the *Weiszfeld method* (Weiszfeld and Plastria, 2009) which is a simple closed form iterative formula. Several studies investigate the convergence of the Weiszfeld method, see e.g., Chandrasekaran and Tamir (1989), Katz (1974), Kuhn (1973), and a number of modifications have been developed, see e.g., Vardi and Zhang (2001), since its first introduction by Weiszfeld in 1937.

In this study, we introduce an extension of the Weber problem so that CO_2 emission amounts are managed by setting the vehicle speeds during the delivery operations of a distribution system. Distribution and transportation related activities are growing all over the world, especially in urban areas, due to a continuous increase in the demand of goods and services (The Population Division of the Department of Economic and Social Affairs of the United Nations, 2016). However, sending vehicles to customers and delivering/picking goods in distribution and transportation logistics result in a significant amount of CO_2 emissions affecting citizens' quality of life and the climate. Thus, the importance of green freight transportation in city logistics has been growing to reduce the harmful effects of CO_2 emission (Demir et al., 2014b).

There are several studies in the literature that take the emission amounts into account mostly for operational and tactical level plannings. For instance, in the context of the traveling salesman problem, the authors in Urquhart et al. (2010b) use an evolution-

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ary algorithm to discover tours with low CO_2 emission. For the vehicle routing problem with time windows (VRPTW), the work in Urquhart et al. (2010a) investigates, using evolutionary algorithms, the trade-off between the number of vehicles used, total CO_2 emission, and the total distance traveled. Up to 10% savings in CO_2 emission amounts are observed in their computational studies. Both of the studies above, take the vehicles' speeds as constant on a given road and do not consider setting the speeds of the vehicles.

Figliozzi in Figliozzi (2011) considers a time-dependent vehicle routing problem which takes the effects of congestion and speeds of the vehicles on the emission amounts into account. Bektaş and Laporte (2011) introduce an extension of the VRPTW named as the pollution routing problem (PRP) which minimizes fuel emission and driver costs. Their solution approach is based on the discretization of the speeds on roads while ignoring a part of the fuel consumption formula in the comprehensive modal emission model (CMEM) (Barth et al., 2005). The ignored part of the formula is shown to be insignificant for speed levels lower than 40 km/h. They formulate the discretized problem as a mixed integer linear programming problem which is able to solve instances of size at most 20 customers within three hours. The authors in Demir et al. (2012) develop an adaptive large neighborhood search algorithm (ALNS) in order to solve the PRP. Their algorithm iterates between two optimization problems: the first one tries to solve the VRPTW using ALNS and the second one solves a speed optimization problem. The bi-objective PRP is studied in Demir et al. (2014a) where the cost of CO_2 emission and the drivers' wages are considered as two conflicting objectives and ALNS is used as the solution procedure along with a speed optimization algorithm. A matheuristic approach for the PRP is proposed in Kramer et al. (2015). Their solution approach is a combination of local search, speed optimization algorithm, and optimization over a set partitioning formulation.

In this study, we consider an extension of the Weber problem, called as the green Weber problem (GWP), which in addition to the location of the single facility also determines the speeds of the vehicles serving the customers from the facility so as to minimize the total amount of CO_2 emitted in the whole distribution system. The customers have one-sided time windows, called the time limits, and the vehicles serving the customers must complete their services within the given time limits. We also consider a time-dependent version of the problem, named as the time-dependent green Weber problem (TD-GWP), in which the vehicle speeds are limited due to congestion. To the best of our knowledge, such extensions of the Weber problem have not been considered before in the literature.

The solution approaches proposed in this study can be used for the strategic level decision of locating a facility for a distribution system prior to making tactical and operational level decisions such as which and what type of vehicles to use and how to route the vehicles.

The rest of the paper is organized as follows: in Section 2, the GWP and the TD-GWP are introduced and formulated as second order cone programming problems. In Section 3, an illustrative example is provided and the solutions of the Weber problem and the GWP, and the solutions of the GWP and the TD-GWP are compared. In Section 4, the results of the computational experiments on large scale randomly generated instances are presented. Finally, we conclude in Section 5 with some future research directions.

2. The green Weber problem (GWP)

We assume that the amount of CO_2 emitted by a vehicle is proportional to its fuel consumption which is aligned with the related literature, see e.g., Demir et al. (2011). As the fuel consumption model, we use the comprehensive modal emission model (CMEM),

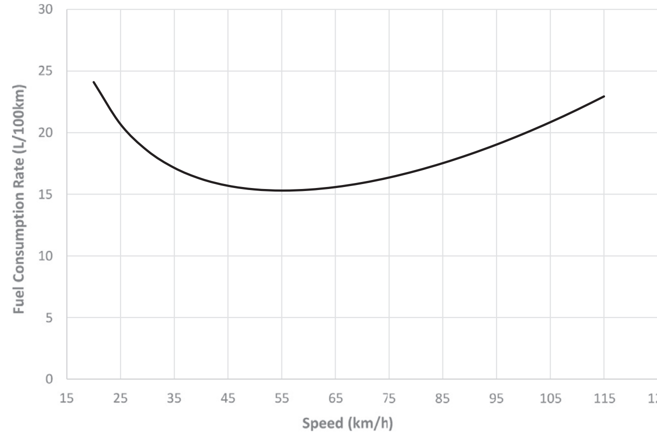

Fig. 1. Fuel consumption in liter per 100 km as a function of the vehicle speed.

Table 1
Values of the parameters used in the fuel consumption calculations.

Notation	Description	Values
λ	Constant	3.09636×10^{-3}
k	Engine friction factor (kilojoule/revolution/liter)	0.2
N	Engine speed (revolution/second)	33
V	Engine displacement (liter)	5
w	Vehicle curb weight (kilogram)	6350
γ	Constant	2.77778×10^{-3}
α	Constant	9.81×10^{-2}
L	Vehicles maximum load (kilogram)	3650
β	Constant	1.64865
c	Fuel and CO_2 emission cost (pounds/liter)	1.4

suggested in Barth et al. (2005), for heavy-good vehicles. For a review and comparison of different vehicle emission models, the reader is referred to Demir et al. (2011). According to the CMEM, the amount of fuel consumed in liter, f, by a vehicle which travels a distance of z km at a speed of v km/h and with a load of L kg is given by

$$f = \lambda kNV\frac{z}{v} + \lambda w\gamma\alpha z + \lambda\gamma\alpha Lz + \lambda\beta\gamma v^2 z, \tag{2}$$

where λ, k, N, V, w, γ, α, and β are the fuel consumption parameters for a particular vehicle type. Fig. 1 shows the fuel consumption rate in liter per 100 km with respect to speed in km/h for a particular vehicle type with no load according to Eq. (2) and is obtained by using the parameter values in Table 1 taken from Demir et al. (2012). As it can be seen from Fig. 1, below the speed of 55.2 km/h, the fuel consumed by the vehicle per unit distance traveled decreases as the speed increases. Above the speed of 55.2 km/h, the fuel consumed per unit distance traveled increases with the speed. Note that the optimal speed, i.e., the speed at which the fuel consumption rate is the smallest, may change from vehicle to vehicle, but for a particular vehicle, it is independent of the load, i.e., constant. Changing the load of the vehicle from L_1 kg to L_2 kg shifts the fuel consumption rate curve by the amount $\lambda\gamma\alpha(L_2 - L_1)$.

Multiplying f with the cost of fuel consumption-CO_2 emission per liter (c), we obtain the cost of fuel-emission, C, for a vehicle that travels a distance of z km at a speed of v km/h and with a load of L kg, as follows

$$C = \alpha_1 z/v + \alpha_2 z + \alpha_3 Lz + \alpha_4 v^2 z, \tag{3}$$

where $\alpha_1 = c\lambda kNV, \alpha_2 = c\lambda w\gamma\alpha, \alpha_3 = c\lambda\gamma\alpha$, and $\alpha_4 = c\lambda\beta\gamma$.

For a fixed distance z, the function in Eq. (3) is convex and the optimal speed minimizing the fuel-emission cost is given by $(\alpha_1/2\alpha_4)^{1/3}$.

In the remaining part of this section, after introducing second order cone programming (SOCP), we formulate the GWP and TD-