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An exact algorithm for the modular hub location problem with single assignments

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ABSTRACT

A key feature of hub-and-spoke networks is the consolidation of flows at hub facilities. The bundling of flows allows reduction in the transportation costs, which is frequently modeled using a constant discount factor that is applied to the flow cost associated with all interhub links. In this paper, we study the modular hub location problem, which explicitly models the flow-dependent transportation costs using modular arc costs. It neither assumes a full interconnection between hub nodes nor a particular topological structure, instead it considers link activation decisions as part of the design. We propose a branch-and-bound algorithm that uses a Lagrangean relaxation to obtain lower and upper bounds at the nodes of the enumeration tree. Numerical results are reported for benchmark instances with up to 75 nodes.

1. Introduction

Hub location problems (HLPs) lie at the heart of network design planning in logistics systems such as the trucking and airline industries. These systems frequently employ hub-and-spoke architectures to efficiently route commodities or passengers between many origins and destinations. Their key feature is the use of transshipment or consolidation points, typically called hubs, to connect a large number of origin/destination (O–D) pairs with only a small number of links. This strategy centralizes handling and sorting operations and reduces set-up costs; most importantly, it makes it possible to achieve economies of scale on routing costs through the consolidation of flows.

HLPs are a challenging class of *NP*–hard combinatorial optimization problems combining location and arc selection decisions. The location decision problem involves the selection of a set of nodes at which hub facilities can be located; the arc selection decision problem addresses the design of the hub network by choosing the links to connect origins, destinations, and hubs, establishing a framework for the routing of commodities through the network. Broadly speaking, the aim of HLPs is to determine the locations of the hubs and the design the hub network so as to minimize the total flow cost (see Alumur and Kara, 2008; Campbell and O'Kelly, 2012; Zanjirani Farahani et al., 2013; Contreras, 2015).

HLPs have received increasing attention since the seminal work of O'Kelly (1986). Analogous to the literature on discrete facility location problems, several classes of HLPs have been studied, including uncapacitated hub location, p-hub median, hub covering, and p-hub center problems. The various applications within each class give rise to variants that differ in terms of assumptions, such as the required topological structure, the allocation pattern of nodes to hubs, and the existence of capacity constraints on the hub nodes or arcs. Nonetheless, there are four common assumptions underlying most HLPs. The first assumption is that commodities have to be routed via a set of hubs, so O–D paths must include at least one hub node. The network induced by the solution of a hub location problem consists of two types of arcs: hub arcs connecting two hubs; and access arcs connecting O–D nodes to hubs. For some applications, in addition to enabling economies of scale, hub facilities may act as consolidation, sorting, and distribution centers. The second assumption is that the hubs can be fully interconnected with more efficient, higher volume pathways that allow a constant discount factor ($0 < \alpha < 1$) to be applied to all transportation costs associated with the commodities that are routed between any pair of hubs. Note that the discount factor is assumed to be independent of the amount of flow that is sent through hub arcs. The third assumption is that hub arcs incur no set-up costs, so hubs can be connected at no extra cost. The forth one is that distances between nodes satisfy the triangle inequality. As a result, the backbone network is typically a complete graph, i.e., the hubs are fully interconnected at no cost.

These assumptions and their implications simplify network design decisions as they are determined mainly by the allocation

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pattern of O–D nodes to hub facilities. As a result, classical HLPs have a number of attractive theoretical features which have given rise to mathematical models that exploit the structure of the network (Alumur et al., 2012; Contreras and Fernández, 2014; Correia et al., 2014; 2010; Ernst and Krishnamoorthy, 1998a; Hamacher et al., 2004; Labbé and Yaman, 2004) and to sophisticated solution algorithms that are able to solve real-size instances (Contreras et al., 2011a; 2011c; Ernst and Krishnamoorthy, 1998b; Labbé et al., 2005; Martins de Sá et al., 2015b).

However, these assumptions may lead to unrealistic results. The independence of flow discounted costs is appropriate in applications in which the links between hubs are associated with faster transportation modes, but it can be an oversimplification in applications where the costs represent the economies of scale due to the bundling of flows on the hub arcs. For instance, full interconnection between hub nodes could lead to solutions in which hub arcs carry considerably less flow than access arcs, yet the transportation costs are discounted only on the hub arcs. It may also be the case that the amounts of flow that are routed on various hub arcs are different, yet the same discount factor is applied across the board. Under the assumption of flow-independent costs and the use of fully interconnected hubs, the overall transportation cost may be miscalculated, and the set of hub nodes selected and the corresponding allocation pattern of O–D nodes to hubs may be suboptimal.

In this paper, we study a *modular hub location problem* (MHLP) which considers explicitly the flow dependence of transportation costs based on modular arc costs. Thus, the total transportation cost is estimated not in terms of the per unit flow cost but in terms of the number of facility links used on each arc, eliminating the use of nonlinear functions and their linearizations to compute the discount factor for each hub arc. The cost is modeled using a stepwise function that determines, for each arc on the network, the total transportation cost as a function of the amount of flow routed through the arc. Our approach can be interpreted in terms of its ability to incorporate multiple capacity levels on the arcs. Another advantage is that it neither assumes a fully interconnected hub network nor a particular topological structure, instead it considers the design of the hub network as part of the decision process. Other variants of MHLP involving multiple assignments and direct connections were initially introduced in Mirzaghafour (2013).

The assumption of modular (or stepwise) transportation costs is consistent with applications in freight transportation and telecommunications networks. In the case of ground transportation, trucking companies send commodities (e.g., goods, express packages, ordinary mail) along hub arcs between break bulk terminals, and along access arcs between an end-of-line terminal and a break bulk terminal, using one or more trucks. The number and capacity of the trucks and the distance traveled can be used to obtain an accurate estimate of the transportation cost between terminals. Here, fixed costs represent the cost of leasing or buying a truck, whereas variable costs may represent the average fuel and labor costs for operating a truck to travel a given distance. The consolidation of flows at hubs allows trucking companies to use large line-haul trucks, typically fully loaded, between hub facilities. Local delivery trucks, typically partially loaded, are used between break bulk and end-of-line terminals to route commodities from origin to destination nodes. Even though both the fixed and variable costs for line-haul trucks are greater than those for local delivery trucks, the per unit transportation cost for hub arcs is lower than that for the access arcs because the trucks have larger capacities. An analogous situation is the use of regional and hub airports by air cargo companies to efficiently route commodities between many origins and destinations. The transportation cost between airports can be estimated based on the number and capacity of the cargo planes, together with the distance.

In the case of telecommunications networks, hub facilities correspond to electronic devices such as multiplexors, concentrators, servers, etc. Commodities correspond to data transmissions that are frequently routed over a variety of physical media (i.e., fiber optic cables, co-axial cables, or telephone lines). The number and capacity of these physical media can be used to provide an estimation of the transmission cost between pair of nodes. Moreover, the modular cost may also represent the usually large set-up cost of the communication links.

Several papers have already pointed out that the discount factor should be regarded as function of the flow volume (see, O'Kelly, 1998; O'Kelly and Bryan, 1998; Bryan and O'Kelly, 1999; Campbell, 2013). O'Kelly and Bryan (1998) were among the first to develop a hub location model that expresses the discount factor on hub arcs as a function of flow. It was later extended by Bryan (1998), Klincewicz (2002) and de Camargo et al. (2009). However, their models use a nonlinear cost function to compute the transportation costs on a hub arc as a function of its flow. This function is approximated by a piecewise linear function to obtain a linear integer programming formulation for the problem. Horner and O'Kelly (2001) proposed a nonlinear cost function based on link performance functions; it is designed to reward economies of scale in all arcs in the network. Podnar et al. (2002) formulated a network design model in which the discount factor applies only on arcs that have flows larger than a given threshold; however, the model focuses on the design of the network rather than on the location of the hub facilities. Racunica and Wynter (2005) introduced a nonlinear concave cost hub location model that determines the optimal location of intermodal freight hubs. The cost function models the flow-dependent discounted cost only on origin-to-hub and hub-to-destination legs.

Yoon and Current (2006) and O'Kelly et al. (2015) adopted a different approach for modeling economies of scale on all the arcs in a hub-and-spoke network. Rather than relying on a nonlinear cost function, they use linear cost functions which combine variable transportation costs for flows on arcs and fixed costs for activating those arcs. In these approaches, the use of fixed costs of arcs allows the link costs per unit of flow to decrease as the flow increases on that link, resulting in the economies of scale. Kimms (2006) presented three different models for hub location problems with fixed and variable costs. He introduced a model, similar to ours, in which the goal is to determine the optimal number of vehicles used on each arc to route flow through the fully interconnected network. However, unlike Kimms (2006), we do not assume a fully interconnected hub network. Cunha and Silva (2007) designed a hub-and-spoke network for a less-than-truckload trucking company in Brazil based on a nonlinear cost function that allows the discount factor on hub arcs to vary according to the total amount of freight between hubs. Campbell et al. (2005a); 2005b) study hub arc location problems, in which the goal is to locate a set of hub arcs, therefore, no longer considering a fully interconnected hub network. To some extent, this mitigates the limitations of flow-independent costs. Other studies consider hub location models focusing on the design of particular topological structures such as star-star networks (Labbé and Yaman, 2008), tree-star networks (Contreras et al., 2009b; Martins de Sá et al., 2013), cycle-star networks (Contreras et al., 2016), and hub line networks (Martins de Sá et al., 2015a; 2015b).

The MHLP is also related to other hub location models where capacities are considered at the arcs of the network. Sasaki and Fukushima (2003) study a capacitated multiple allocation HLP where capacity constraints are considered both on hub nodes and hub arcs. However, in their model, the flow between each O-D pair can go through at most one hub facility and hence there is no discount between hubs. Yaman and Carello (2005) introduce the capacitated single assignment hub location problem with modular