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# Routing and scheduling decisions in the hierarchical hub location problem

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ABSTRACT

Hubs are facilities that consolidate and disseminate flow in many-to-many distribution systems. The hub location problem considers decisions that include the locations of hubs in a network and the allocations of demand (non-hub) nodes to these hubs. We propose a hierarchical multimodal hub network structure, and based on this network, we define a hub covering problem with a service time bound. The hierarchical network consists of three layers in which we consider a ring-star-star (RSS) network. This multimodal network may have different types of vehicles in each layer. For the proposed problem, we present and strengthen a mathematical model with some variable fixing rules and valid inequalities. Also, we develop a heuristic solution algorithm based on the subgradient approach to solve the problem in more reasonable times. We conduct the computational analysis over the Turkish network and the CAB data sets.

## 1. Introduction

Hubs function as switching, transshipment and sorting points in many-to-many distribution networks. Instead of connecting each origin-destination (o-d) pair by a direct link, hubs provide a connection between each pair by using fewer links and concentrating demand flows to allow economies of scale.

The hub location problem is to decide on the locations of hubs and the allocations of demand nodes to hubs. Versions of the hub location problem are defined as ‘single allocation’ and ‘multiple allocation’. In a single-allocation hub network, each demand node is assigned to exactly one hub. Whereas, in a multiple-allocation hub network, demand nodes can be allocated to more than one hub. The classic hub location problem has three main assumptions. First, the hub network is assumed to have a complete structure, with a link between each hub pair. Second, there are economies of scale between hubs. Third, direct transportation between demand node pairs (without using any hubs) is not allowed (Campbell, 1994).

Main application areas of the hub location problem are cargo delivery, telecommunications network design and air transportation. In this study, we mainly focus on a cargo delivery application in Turkey with a given service time promise. A classic cargo delivery system consists of branch offices and operation centers. Branch offices collect and distribute cargoes from/to customers directly, and operation centers collect and distribute cargoes from/to branch offices or send cargoes to another operation center. Although there can be more than one branch office in a city, operation centers do not exist in every city. Thus, each branch office must be assigned to operation center(s).

According to the above explanation of a cargo delivery system, cargo delivery networks and hub location networks are very similar. Branch offices and operation centers in cargo delivery networks can be considered as demand nodes and hubs, respectively. Also, there are economies of scale due to bulk transportation between operation centers. Therefore, the cargo delivery problem can be considered as a hub location problem (Alumur and Kara, 2009; Kara and Tansel, 2001; Tan and Kara, 2007; Yaman et al., 2007; 2012 and Alumur et al., 2012b). In Kara and Tansel (2001), the authors emphasize the importance of synchronization in cargo delivery systems. Later, Yaman et al. (2012) combine the release time scheduling and hub location problems in cargo delivery applications.

The classic hub location problem proposed by O’Kelly (1986a), 1986b), 1987) only considers minimization of the total transportation cost. However, in real life, cargo companies pay similar attention to customer satisfaction. To attract more customers, cargo companies focus on service levels. Service level in cargo delivery is usually measured by delivery time Yaman et al. (2012). Reducing delivery time is generally considered to increase customer satisfaction, and thus cargo companies offer different delivery time promises. For instance, in Turkey, cargo companies aim to deliver cargoes within 24 hours (next-day delivery). However, due to

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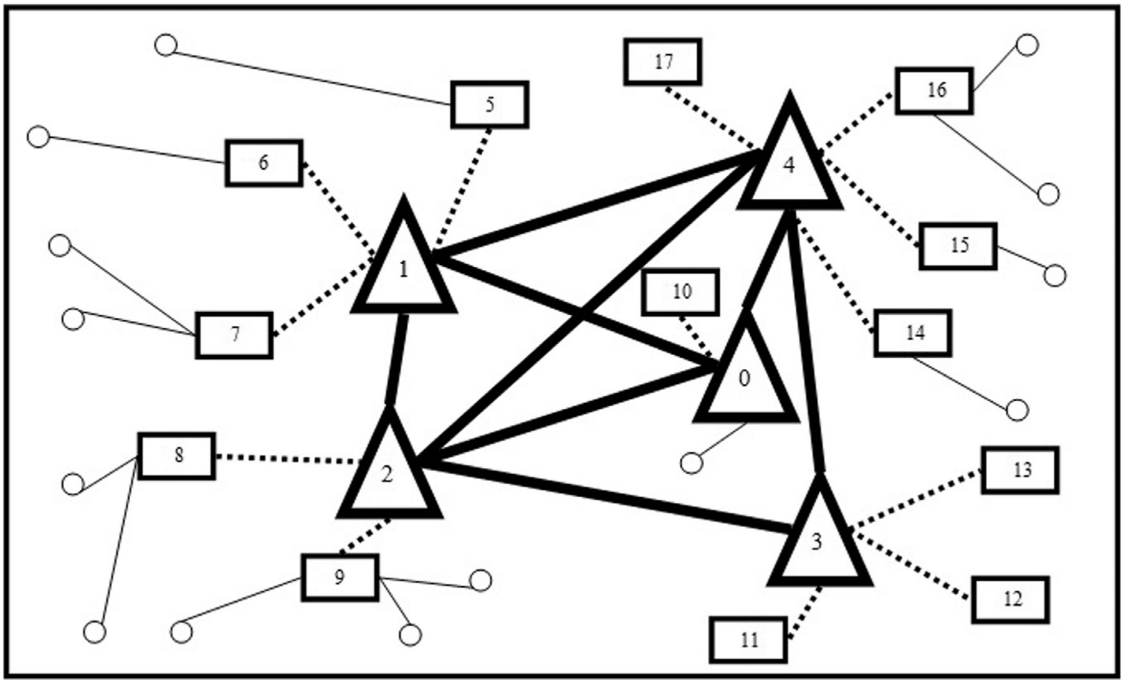

**Fig. 1.** Representation of a Multimodal Hierarchical Hub Network Instance.

Turkey's geographical structure, delivery within this time frame using ground transportation is almost impossible for some city pairs. Thus, in order to keep the next-day delivery promise between all city pairs, cargo companies in Turkey have begun to use airplanes in their distribution networks.

In the classic hub location problem, the hub network (the subnetwork that is induced by the hub nodes and links between them) is usually assumed to be complete with a link between each hub pair. When using airplanes, a complete hub network results in many flights, and operating a flight is very costly. Customarily, the operational cost of a flight consists of a fixed dispatch cost and a variable transportation cost, which depends on the length of the flight. In this research, we limit our study to consider only the fixed dispatch cost and to those cases where that cost is significantly more crucial than the variable transportation cost. We have two main motivations for this limitation. First, the fixed dispatch cost consists of crucial cost components of a flight such as taxiing and take off/landing costs, which are common for each flight not depending on the length of flight. Second, since we consider a real cargo delivery application in Turkey, the variable transportation cost is not as important as the fixed dispatch cost due to the geographical structure of Turkey in which the difference among distances between any two airport hub candidates can be negligible compared to other countries that has bigger land area such as USA. Based on these motivations and reasons, we set the main goal as minimizing the number of airline segments (flights). Thus, we want to provide the same worst-case level of service to all o-d pairs with minimal number of flights.

Motivated by the cargo company that uses trucks (small and large) along with airplanes in their network, we consider a hierarchical multimodal network with three layers and two types of hubs (ground and airport). Fig. 1 shows such an instance with 18 hubs; nodes 0 to 4 are airport hubs; nodes 5 to 17 are ground hubs and the small circles with no numbers represent the demand points. In this representation, airline segments are illustrated as thick lines between the airport hubs; highway segments are illustrated as thin lines between demand nodes and hubs (ground or airport), and as dashed lines between ground hubs and airport hubs.

The lowest layer of the network consists of the allocations of the demand points to the ground hubs and airport hubs (thin solid lines) as necessary to meet the single allocation. In this layer, a star structure is used to allocate the demand points. Each demand node is connected to exactly one hub (ground or airport) with a highway link. In real life, small trucks are used on these highway segments.

The middle layer includes the allocation of ground hubs to airport hubs (dashed lines), and we consider a star structure to allocate the ground hubs here as well. Each ground hub is connected to exactly one airport hub with a highway link. Large trucks, which are faster and have more capacity than small trucks, are assumed to be used on these highway segments, and thus economies of scale are considered.

Fig. 1 depicts a mesh structure in the first (top) layer (thick lines), where airport hubs are connected with each other via an airline segment. However, to accomplish the fundamental goal, that is, to decrease the number of flights, we propose using a ring structure in the top layer instead of a mesh structure, which can cause more flights (Fig. 2). We call this type of network a ring-star-star (RSS). We assume each ring will be served by separate airplanes. To cope with synchronization issues, routing and scheduling decisions must be considered together.

With the ring structure in the top layer, the airplane route is decided while covering all o-d pairs within a given time bound. In this study, motivated by the cargo company's application, we adopt a "pick up, then deliver" type of service, which means there are two separate tours; a pick-up tour and a delivery tour. In the pick-up tour, all demands are collected from their origins and sent to a specific airport hub. After all demands arrive at this airport hub, they are sent to their final destinations in the delivery tour. We need a specific airport hub to collect all the demands at one point, and we call this the central airport hub. In Fig. 2, we denote the central airport hub with a big circle. If one airplane in a ring is not enough to cover all o-d pairs within the time limitations, there can be more than one ring, as shown in Fig. 2. In each ring, exactly one airplane can travel because there is no capacity restriction.

Pick-ups from the origins to the central airport hub and deliveries from the central airport hub to the destinations are assumed