Computers and Operations Research 89 (2018) 31–50

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Benders decomposition applied to a robust multiple allocation incomplete hub location problem

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ARTICLE INFO

Article history:
Received 11 August 2016
Revised 31 July 2017
Accepted 1 August 2017
Available online 4 August 2017

Keywords:
Hub network design problem
Robust optimization
Benders decomposition method
Demand uncertainty
Fixed setup cost uncertainty
ILS-VND Heuristic

ABSTRACT

This paper focuses on a multiple allocation incomplete hub location problem in which a hub network can be partially interconnected by hub arcs, direct connections between non-hub nodes are allowed, and uncertainty is assumed for the data of origin-destination demands and hub fixed costs. This problem consists of locating hubs, activating hub arcs and routing the demand flows over the designed network such that the total cost is minimized. The total cost is composed of fixed setup costs for hubs and hub arcs, and of transportation costs. This problem has economical and social appeals for designers of public transportation systems and other hub networks. A robust optimization approach is chosen to address the data uncertainty considering that demand flows and fixed setup costs are not known with certainty in advance. The computational experiments on benchmark instances from the hub location literature showed that the proposed robust model renders better assurance of not violating budget constraints than the deterministic version. Further, two specialized Benders decomposition frameworks and an ILS-VND stochastic local search procedure are also devised to tackle larger problem instances with up to 100 nodes in reasonable computational times.

1. Introduction

The intense exchange of entities (e.g. people, freight, data) among different regions impels the need for more efficient forms of connecting these areas to better route the flows. Among the available forms of networking, hub-and-spoke networks detach themselves by their widely presence on telecommunication and transportation systems, and on postal services. A hub-and-spoke network is a type of network architecture which can arise whenever many origin and destination pairs of nodes exchange flows. Instead of directly routing flows from their origin node to their destination node, flows are routed through nodes selected to act as hubs.

Hubs are special nodes of the network responsible for receiving, aggregating, and routing the flows, either to their final destinations or to other hubs, which for their turn will redirect the flows, either to other hubs or will disaggregate the flows to send them to their final destinations. Hubs are inter-connected by hub arcs forming a hub level network, while non-hub nodes are assigned to hubs by means of allocation links, composing an access or tributary network. Since demand flows are routed in bulks through a hub level network, high-capacity carriers can be used on hub arcs so that lower unitary transportation costs can be attained and consequently, scale economies can be exploited.

Fig. 1 illustrates an incomplete multiple allocation hub-and-spoke network. Gray circles and bold triangles represent nodes exchanging flows and hubs, respectively. Hubs are interconnected by hub arcs (bold links) forming a hub level network; while non-hub nodes may be allocated to more than one hub by allocation links (thin links), composing an access or tributary network. Furthermore, direct connection between non-hub nodes are allowed (dotted links). Notice that the hub level network is incomplete, i.e it does not have all the possible hub arcs installed.

Generally speaking, a hub-and-spoke network design problem (HNDP) consists of selecting some nodes from a set of candidate nodes to become hubs, deciding which hub arcs will be installed and granted scale economies, and allocating non-hub nodes to hubs such that a given objective is optimized (O'Kelly and Miller, 1994). These general design guidelines are aligned to the classification scheme of hub-and-spoke topologies introduced by O'Kelly and Miller (1994). This classification of topologies gives rise to a

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http://dx.doi.org/10.1016/j.cor.2017.08.001
0305-0548/

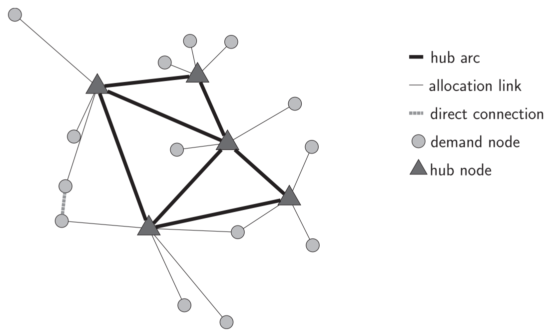

Fig. 1. Illustration of an incomplete multiple allocation hub-and-spoke network allowing direct connection between pair of non-hub nodes.

set of eight protocols which classify hub networks by observing three important topological characteristics of a network.

The first one concerns with the allocation of non-hub nodes to hubs. Allocations can be of the single type in which each node is assigned to a single hub only; or of the multiple type in which a node can be connected to more than one hub. The second one relates to the consent of direct connections among non-hub nodes. Finally, the third feature concerns with the inter-connection of the hub level network. The hub level network can be fully interconnected with a hub arc between each pair of hubs or partially inter-connected being often referred to as an incomplete hub network.

In order to design hub networks, different objectives have been addressed in the hub literature. Among these objectives, the most common are the minimization of: (*i*) the total cost which, in general, is composed by the sum of transportation costs for routing the flows between all origin and destination nodes, fixed setup costs for establishing the hub nodes and for installing the hub arcs (Alumur et al., 2009; O'Kelly, 1987; 1992); (*ii*) some performance measure associated with time, such as the latest arrival time (Kara and Tansel, 2001) or the average travel time (Martins de Sá et al., 2015a); (*iii*) the total cost for establishing hubs and hub arcs such that a given pattern of coverage is guaranteed as, for instance, ensuring that the total travel time for routing each origin-destination demand meets a given time limit (Alumur and Kara, 2009; Calık et al., 2009; Campbell, 1994), and (*iv*) the maximum transportation cost or the maximum travel time (Campbell, 1994).

Over the years the HNDP had its assumptions broadened since the problem was first introduced by O'Kelly (1986); 1987). Originally O'Kelly has assumed that hubs were fully interconnected by hub arcs. This is a reasonable assumption whenever the installation of hub arcs is cost-free or the costs to install these kind of links are irrelevant compared with others cost parameters. On the other hand, there are situations in which a complete hub level network is impracticable due to the probably prohibitive costs of activating all hub arcs, the low flow levels on some hub arcs that do not justify the use of a large low unit cost vehicle or due to some insurmountable geographical related issue preventing the setup of some hub arcs (O'Kelly and Miller, 1994). Hence for these cases, more flexible hub network designs have been investigated by the hub-and-spoke research community bringing about many different types and variants of the problem. For thorough surveys on the many available extensions for the HNDP, please refer to Farahani et al. (2013), Campbell and O'Kelly (2012), Alumur and Kara (2008), Campbell et al. (2002), and Klincewicz (1998).

This paper focuses on a multiple allocation HNDP in which the hub network can be partially interconnected, and direct connections linking non-hub nodes are allowed. The total cost is composed of fixed setup costs for hubs and hub arcs, and of transportation costs for routing demand flows. The problem here addressed has a great appeal for designers of public transportation systems (Gelareh and Nickel, 2011). This problem consists of locating hubs and of activating hub arcs such that the total cost is minimized. A similar problem of designing a multiple allocation incomplete hub network, where direct connections between non-hub nodes are not allowed, was addressed in Gelareh and Nickel (2011). Martins de Sá et al. (2015a) also addressed a similar problem of designing a multiple allocation incomplete hub network, but allowing direct connections between non-hub nodes and ensuring that the hubs are connected through lines. In these studies, parameter uncertainty and robustness were not considered.

In these systems, the hub-and-spoke network can be seen as a multi-modal network composed of high capacity transportation modes (e.g. bus rapid transit or metro) and low capacity transportation modes (e.g. bus lines, taxicabs or walking). Hubs act as inter-modal nodes being responsible for changes of modes and for transferring passengers by means of high capacity vehicles for achieving economies of scale; while low capacity modes are used in the access network. These low capacity modes can directly route flows between pairs of non-hub nodes or between pairs of non-hub and hub nodes. For instance, in the design of a metro system, network planners are interested in where to locate the metro stations (the hubs) and where to install the railways connecting pairs of stations. Fixed costs for hubs arise in the installation, maintenance, and operation of the metro stations, while fixed costs for hub arcs can be the costs of establishing and maintaining the railway line that connects the stations. These costs can be composed of costs associated with land expropriations, tunnel excavations for an underground subway, materials, and manpower for construction and maintenance and so on.

For public transportation systems, the multiple allocation and the direct connections policies are acceptable assumptions. A given origin node can reach different hubs (stations) based on the desired destination node; while origin-destination demand nodes can be directly connected by an alternative transportation mode (e.g. bus lines, taxicabs or walking) which does not pass through or rely on any hub (station). These more flexible assumptions allow a network design to fit better to the application being addressed. Nevertheless the efficiency of the designed network still greatly relies on the quality of the data parameters used during the project conception.

An accurate prediction of cost structures for locating hubs, for installing hub connections or for routing flows may be a formidable task, which is usually overcome in hub location problems by estimating the parameter values by means of data forecasting. When these values are poorly estimated, profound impacts on the real total costs may occur. Due to the high costs of installing infrastructure, a redesign of the proposed network may be out of the question. It is thus important to take into account uncertainty for the data parameters when designing a hub network. Here, a robust optimization approach is adopted to handle the data uncertainty for the HNDP.

In this work, the uncertainty is assumed for two data parameters: for the origin-destination demands and for the fixed setup costs. Demand uncertainty can arise due to the combination of several random factors (variables), e.g. weather changes, economical fluctuations, and the availability of alternative transportation systems. These factors can make the future demand poorly predictable. Likewise the uncertainty of the fixed setup costs can happen due to other random variables, e.g. labor strikes, supply rupture, new environmental policies, political or geographical issues. For instance, in the design of a metro, accurately measuring the exact costs of installing railway lines in regions with mountainous topography (for elevated railways) or other geological situations is not always possible during the design of the network, being therefore subject to have the costs revised during construction