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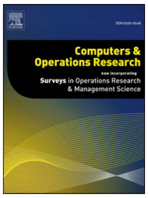

# Multi-commodity location-routing: Flow intercepting formulation and branch-and-cut algorithm

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ABSTRACT

Research on the location-routing problem (*LRP*) is very active, producing a good number of effective exact and approximated solution approaches. It is noteworthy that most of the contributions present in the literature address the single-commodity *LRP*, whereas the multi-commodity case has been scarcely investigated. Yet, this issue assumes an important role in many *LRP* applications, particularly in the context of designing single-tier freight distribution City Logistics systems. To fill this gap, we define a new multi-commodity *LRP*, proposing an original integer linear programming model for it. The proposed formulation takes into account the multi-commodity feature of the problem, modeling the strategic location and the tactical routing decisions using the flow intercepting approach. We therefore name this problem the flow intercepting facility location-routing problem. It is solved by a branch-and-cut algorithm which exploits cuts derived and adapted from literature. The proposed method is successfully experienced and validated on test instances reproducing different network topologies and problem settings.

## 1. Introduction

The literature on the location-routing problem (*LRP*) has significantly increased in the last ten years, even though the first contribution on the theme dates back to Maranzana (1964). In its basic version, it can be defined as follows. Given a set of potential facility locations and a set of customer demands to be satisfied, we have to simultaneously determine: the number and position of one or more facilities (strategic location decisions); the customer-to-facility (one-to-one) assignment (strategic assignment decisions); the size of the vehicle fleet used to serve the customer and the routes to be performed by each vehicle dispatched from the located facilities (tactical routing decisions). The aim is the minimization of the total system costs, given by the sum of location and distribution costs.

*LRP*, related variants and applications have been largely addressed through exact and approximated approaches, as illustrated by the recent surveys of Prodhon and Prins (2014), Drexl and Schneider (2015) and Cuda et al. (2015). Yet, to the best of the authors' knowledge, most of the *LRP* contributions present in literature deal with single-commodity flows, whereas the multi-commodity case has been scarcely investigated. This issue assumes a relevant role in many *LRP* applications, in particular, the ones arising in City Logistics, *CL*, (Bektas et al., 2016; Crainic et al., 2009; Mancini et al., 2014). Indeed, the goods/service demand to be managed in an urban area is highly customized. Hence, the corresponding distribution problem is strongly and inherently a multi-commodity flow problem.

This work is aimed at filling this gap on the multi-commodity *LRP* in a *CL* perspective. The design of a single-tier urban freight distribution system is the driving application. Hence, for the sake of clarity, we briefly recall the single-tier basic idea (Taniguchi et al., 1999; Crainic et al., 2004), highlighting the related design issues to be integrated in a multi-commodity *LRP*. Then, we focus on the proposed methodological aspects.

The aim of a single-tier system is to prevent the penetration of a large number of commercial vehicles, coming from the primary logistic facilities located on the city outskirts and directed to the city center, stopping them at secondary or intermediate logistic facilities (Boccia et al., 2011). Here the freight flows of different carriers are deconsolidated, transferred and consolidated into smaller and green vehicles, more suitable to perform the delivery to the final customers. This allows to reduce the vehicle kilometers traveled and to remove vehicles from the urban network. Such systems

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have already found their applications in several urban areas, e.g., Munich, Siena and Padova, where large size vehicles are forbidden to enter the city center and final distribution is performed by freight distribution companies (Crainic et al., 2009; Morana, 2014).

In this context the *LRP* consists in determining, at the same time, the number and the location of the intermediate logistic facilities and the routes to be performed by the vehicles to supply the customer demands, minimizing the total system cost. Moreover, two other important issues should be also properly taken into account to realize *CL* solutions acceptable and convenient for all the stakeholders (public authorities, goods/service companies, freight carriers and final customers). First, the customer-to-facility assignment should be made not only considering the position of the final customer, but also the origin of the commodity required by the customer. This makes the distribution system more coherent with traditional flows and used corridors and, moreover, avoids long trips to reach the intermediate logistic facilities and/or the final customers, so impacting on the delivery time. Second, in many cities, and in particular in smaller ones, the choice of the logistic facility locations is strongly conditioned by the urban infrastructure and road system. Hence, they should be located in correspondence of pre-existing structures (e.g. available parking areas) and, when possible, along or near the main entrance roads, consistent with the vehicular travel demand pattern. This, on one side, allows reducing the installation costs, on the other side, makes the *CL* measure well-accepted by the stakeholders.

On this basis, the main goals of this paper can then be stated as: (1) Defining a multi-commodity *LRP* integrating the just described *CL* design issues; (2) Proposing an original integer linear programming (ILP) formulation for the problem using a flow-intercepting approach for the location decisions; and (3) Proposing an exact solution approach and benchmark instances for this new problem.

The second goal requires a preliminary discussion. *LRP* formulations present in literature are generally obtained merging the ILP models of the two sub-problems composing it, i.e., facility location and vehicle routing. The routing sub-problem is approached with classical path or arc based formulations (a slightly modified arc based formulation is used in this paper). The location problem has been addressed as a point-based demand facility location problem, generally adapting the classical p-median or simple plant facility location formulations. In this work, we treat the location decisions as a flow-based demand facility location problem (Zeng et al., 2010; Sterle et al., 2016), i.e., as a location problem where facilities do not generate or attract flows, but intercept them along their pre-planned paths from their origins to their destinations (Hodgson, 1981; Berman et al., 1992; Boccia et al., 2009). In literature, this path coverage problem is referred to as flow intercepting facility location problems, *FIFLP*. The choice of using *FIFLP* in our *LRP* formulation is motivated by the fact that a path-based approach, including information about origin and destination of each commodity, naturally fits with multi-commodity flows and allows to easily integrate the discussed *CL* issues. Indeed, even if the point-based formulation can be modified for more commodities, this generates a significant increase in the size of the problem, making it unsolvable in a reasonable time.

For this reason, we call *flow-intercepting facility location-routing* problem, *FIFLOR*, the proposed multi-commodity *LRP* formulation. To the best of authors' knowledge, it has never been treated before. *FIFLOR* will be optimally solved by a branch-and-cut algorithm based on valid inequalities derived and adapted from the literature. A heuristic procedure for the upper bound (*UB*) computation, exploiting the solution of the linear relaxation of the *FIFLOR* formulation, will also be presented. The proposed *LRP* approach has been experienced and validated on several test instances representing different scenarios of the single-tier freight distribution design problem.

The paper is structured as follows: Section 2 recalls the main contributions on *FIFLP* and multi-commodity *LRP*, in order to position the *FIFLOR* in the literature. Section 3 presents an original ILP formulation for the problem. Section 4 describes the branch-and-cut algorithm and the heuristic procedure. Finally, we present and discuss numerical results in Section 5.

## 2. Literature review

In the following, we provide a short review about the *FIFLP* and the multi-commodity *LRP*.

The literature on *FIFLP* is rather scarce compared to that of classical point-based facility location problems. To the best of authors' knowledge, *FIFLP* has never been used before in the *CL* context. Reviews of the main contributions can be found in Berman et al. (1995), Boccia et al. (2009) and Zeng et al. (2010). Contributions can be cited in several fields including: urban traffic management (counting sensors, cameras and variable message signs; Gentili and Mirchandani, 2005; Yang et al., 2006; Sterle et al., 2016); park-and-ride (Horner and Groves, 2007); flow control/monitoring for security (inspection stations and checkpoints; Gendreau et al., 2000; Selmic et al., 2010; De Cillis et al., 2013); logistics (convenience stores and refueling stations: Upchurch et al., 2009; Kim and Kuby, 2013; Wen et al., 2014). This brief list confirms the wide applicability of the *FIFLP* approach in different contexts. However, the solution methodologies proposed for these applications cannot be easily generalized and transferred from an application to another, because they all take into account practical constraints which are typical of the specific problem under investigation.

Concerning *LRP*, a review of the first papers on the theme can be found in Laporte (1988) and Nagy and Salhi (2007). For the most recent contributions the interested reader is addressed to the surveys by Prodhon and Prins (2014), Drexl and Schneider (2015) and Cuda et al. (2015). The first starts with a review of the basic *LRP*, its formulations and solving approaches and then reports on main contributions for several extensions (e.g., multi-objective, truck-and-trailer, stochastic formulations, etc.). The second can be considered as complimentary to the first, since it provides a deeper insight on main *LRP* variants and extensions. Finally, the third focuses on the two-echelon vehicle routing problems, with or without location decisions (*2E-VRP* and *2E-LRP*). Hence, the three reviews, even if presenting some overlapping, complement each other and provide a wide and detailed review of all the recent methodological advances and applications of *LRP*. As will be clarified in the following, *FIFLOR* can be interpreted as both a particular case of the *LRP* or *2E-LRP*.

These reviews demonstrate that multi-commodity *LRP* has been scarcely treated in the literature. To the best of authors' knowledge, the first work explicitly tackling multi-commodity *LRP* is the one by Burks (2006). The author adapted and integrated the ILP model proposed by Perl and Daskin (1985) and presented an ILP formulation solved by a tabu search metaheuristic. Similar developments were proposed by Hamidi et al. (2012, 2014), where a multi-commodity *LRP* was modeled by an ILP formulation and solved by metaheuristic and local search approaches. Recently, Rath and Gutjahr (2014) defined a multi-commodity *LRP* in the context of disaster relief. Yet, assuming the goods to be distributed as homogenous, they formulated the problem as a single commodity *LRP*, solving it by a matheuristic. Giannessi et al. (2015) addressed a particular variant of the multi-commodity *LRP*, where the facilities to be located have to be connected via a ring. The authors proposed an ILP formulation and solved the problem by an exact method, a matheuristic and a hybrid approach. Finally, Rahmani et al. (2015a, 2015b) addressed a particular variant of the