Computers and Operations Research 83 (2017) 1–12

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

An exact hybrid method for the vehicle routing problem with time windows and multiple deliverymen

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A R T I C L E I N F O

Article history:
Received 20 June 2016
Revised 29 December 2016
Accepted 1 February 2017
Available online 2 February 2017

Keywords:
Vehicle routing
Multiple deliverymen
Hybrid method
Branch-price-and-cut
Metaheuristics
Column generation

A B S T R A C T

The vehicle routing problem with time windows and multiple deliverymen (VRPTWMD) is a variant of the vehicle routing problem with time windows in which service times at customers depend on the number of deliverymen assigned to the route that serves them. In particular, a larger number of deliverymen in a route leads to shorter service times. Hence, in addition to the usual routing and scheduling decisions, the crew size for each route is also an endogenous decision. This problem is commonly faced by companies that deliver goods to customers located in busy urban areas, a situation that requires nearby customers to be grouped in advance so that the deliverymen can serve all customers in a group during one vehicle stop. Consequently, service times can be relatively long compared to travel times, which complicates serving all scheduled customers during regular work hours. In this paper, we propose a hybrid method for the VRPTWMD, combining a branch-price-and-cut (BPC) algorithm with two metaheuristic approaches. We present a wide variety of computational results showing that the proposed hybrid approach outperforms the BPC algorithm used as standalone method in terms of both solution quality and running time, in some classes of problem instances from the literature. These results indicate the advantages of using specific algorithms to generate good feasible solutions within an exact method.

1. Introduction

Vehicle routing problems have been widely studied over the last decades due to practical relevance as well as their challenging formulations (Irnich et al., 2014). In this paper, we address the Vehicle Routing Problem With Time Windows and Multiple Deliverymen (VRPTWMD), an extension of the Vehicle Routing Problem with Time Windows (VRPTW) recently introduced in the literature (Pureza et al., 2012). In the VRPTWMD, the service time at the operational points (e.g., customers) of a route depends on the crew size assigned to that route. Therefore, in addition to the typical scheduling and routing decisions, the VRPTWMD treats the number of deliverymen assigned to each route as a decision variable.

The VRPTWMD is motivated by the real-life delivery and/or pick-up of goods in congested urban areas where promptly available parking spaces near customers' locations can be very limited. To deal with this problem, nearby customers are considered as a single cluster and a common parking location is chosen for the customers in the cluster. Then, the vehicle parks at this common location and the workers deliver (and/or collect) the products on foot to all customers in the cluster. Because of this operational mode, service times at the clusters can be very long, usually corresponding to a large percentage of the total route duration. This can compromise the service of all customers during regular working hours, especially when only one deliveryman is assigned to the routes. In this context, using additional deliverymen becomes relevant, as a larger number of deliverymen in a route leads to shorter service times at each cluster of customers.

In this paper, we propose a hybrid method based on a cooperative scheme between a branch-price-and-cut (BPC) algorithm and two metaheuristic approaches for the VRPTWMD. The metaheuristic approaches are based on Iterated Local Search (ILS) and Large Neighborhood Search (LNS) respectively, and use tailored operators within the search. The BPC algorithm uses state-of-the-art features such as an interior point stabilization strategy in the column generation process, separation of valid inequalities using well-centered points of the feasible sets, strong branching and a MIP-based primal heuristic. In the hybrid method, metaheuristics can improve the capacity of the BPC to generate good feasible solutions at an earlier stage while retaining the advantage of the BPC in providing good lower bounds.

There are few papers proposing solution methods for the VRPTWMD. Pureza et al. (2012) model this problem based on the

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http://dx.doi.org/10.1016/j.cor.2017.02.001

standard arc flow formulation for the VRPTW (Desaulniers et al., 2014). As current general-purpose integer programming solvers are not effective to solve such formulations within a reasonable running time, the authors propose two metaheuristic approaches to solve the problem. The first one is a Tabu Search (TS) algorithm characterized by an adaptive mechanism that changes the parameters of the search based on an analysis of the search trajectory pattern. The second solution approach is an Ant Colony Optimization (ACO) algorithm that constructs solutions through a probabilistic insertion mechanism. Senarclens de Grancy and Reimann (2016) propose two metaheuristic algorithms based on ACO and GRASP to solve the VRPTWMD. To make the algorithms comparable, the metaheuristics rely on the same constructive heuristic and local search components. Álvarez and Munari (2016) propose two metaheuristic approaches based on ILS and LNS for the VRPTWMD. In addition to the typical components of these metaheuristics, both approaches use a set of specific heuristics designed to enhance its performance, namely, route reduction heuristic and deliverymen reduction heuristic. The latter approaches are the current state-of-the-art metaheuristic algorithms for the VRPTWMD. Ferreira and Pureza (2012) develop two heuristic algorithms for a variant of the problem without time windows and drop the requirement of visiting all customers. In their study, the first algorithm is an extension of the savings algorithm (Clarke and Wright, 1964) and the second is a TS heuristic that uses an adaptive mechanism as in Pureza et al. (2012). Munari and Morabito (2016) propose a BPC method for the VRPTWMD, which is the first exact algorithm proposed specifically for this problem. As reported by the authors, the method was able to find optimal solutions for the first time for several instances. Moreover, improved upper bounds were obtained by the method.

Although the VRPTWMD can be used as basis for practical and theoretical applications, to the best of our knowledge, no hybrid methods have been proposed in the literature to solve it. Hence, the main contribution of this paper is that we develop a hybrid method to solve it, reporting computational results obtained on instances from the literature. In addition, we provide experimental results to assess the benefits of using extra deliverymen in scenarios allowing different limits on the crew size. In the past several years, hybrid methods combining exact and metaheuristics approaches have become popular for solving vehicle routing problems (Alvarenga et al., 2007; Cacchiani et al., 2014; Danna and Le Pape, 2005; Kramer et al., 2015; Nishi and Izuno, 2014; Salari et al., 2010; Subramanian et al., 2012; Villegas et al., 2013). These hybrid methods attempt to simultaneously exploit the advantages of their components to obtain better solutions than those obtained by those same components as standalone approaches.

The remainder of this paper is organized as follows. In Section 2, we describe the VRPTWMD and a set partitioning formulation for the problem. The hybrid method introduced to solve the VRPTWMD and its components are detailed in Section 3. The results of our computational experiments are described in Section 4. Finally, in Section 5, we present the main conclusions of this research.

2. Problem description

In the VRPTWMD, an optimal solution must satisfy the constraints of vehicle capacities, time windows and available deliverymen while minimizing the total cost composed by the number of vehicles, number of deliverymen and traveled distance costs of the routes. Similar to previous studies on this problem (Álvarez and Munari, 2016; Senarclens de Grancy and Reimann, 2016; Munari and Morabito, 2016; Pureza et al., 2012), we assume that the clusters of customers are defined in advance, thus, decisions concerning the selection of parking locations and customers belonging to the clusters are input data. These data also include service times for the clusters as a function of the number of deliverymen that can be assigned to a vehicle.

Consider a set of identical vehicles initially located at a central depot. Each can only perform a single route. A solution to the problem consists of a set of routes that start and end at the depot such that the demand of each cluster is served by exactly one vehicle (no splitting allowed) and within the cluster time window. Each route must have a defined number of deliverymen that is maintained throughout the route. Therefore, given a solution S, its cost is defined by

$$c(S) = w_1 v + w_2 d + w_3 t \tag{1}$$

where v is the number of vehicles used, d is the total number of deliverymen assigned and t is the total traveled distance in S. Moreover, w_1, w_2 and w_3 are the weights of each objective, which can be used to define the priorities of these objectives.

2.1. A set partitioning formulation for the VRPTWMD

The VRPTWMD can be formulated as a set partitioning (SP) model (Munari and Morabito, 2016). Let L be the maximum number of deliverymen that can be assigned to a single route, D be the total number of available deliverymen at the depot and n be the number of customer clusters. Also, let P^l be the set of all feasible routes using l deliverymen, $l = 1, 2, \dots, L$. We say that a vehicle travels in mode l if its crew size is equal to l. We associate the following parameters with each route $p \in P^l$: c_p^l represents its cost and a_{pi}^l, $i = 1, \dots, n$, a binary coefficient taking the value 1 if and only if route p serves cluster i in mode l. Finally, let λ_p^l be a binary variable that is equal to 1 if and only if route p is selected. Using this notation, the VRPTWMD can be formulated as follows:

$$\min \sum_{l=1}^{L} \sum_{p \in P^l} c_p^l \lambda_p^l \tag{2a}$$

$$\text{s.t.} \sum_{l=1}^{L} \sum_{p \in P^l} a_{pi}^l \lambda_p^l = 1, \quad i = 1, \dots, n, \tag{2b}$$

$$\sum_{l=1}^{L} \sum_{p \in P^l} l \lambda_p^l \le D, \tag{2c}$$

$$\lambda_p^l \in \{0, 1\}, \quad l = 1, \dots, L, \; p \in P^l. \tag{2d}$$

The objective function (2a) minimizes the total cost of the selected routes. Following the definition stated in (1), the cost of each single route p that sequentially visits the clusters $i_0, i_1, \dots, i_r, r > 0$, is given by

$$c_p^l = w_1 + w_2 l + w_3 \sum_{j=0}^{r-1} d_{i_j i_{j+1}}, \tag{3}$$

where d_{ij} is the Euclidean distance between clusters i and j. Constraints (2b) specify that each cluster i must be visited by exactly one vehicle in one single mode l. Constraint (2c) imposes the total number of available deliverymen. Finally, the binary requirements on the variables are imposed by constraints (2d).

A lower bound on the optimal value of (2a)–(2d) can be obtained by solving its linear programming (LP) relaxation. Indeed, the main advantage of this formulation is that it has a stronger LP relaxation than the arc flow formulation proposed by Pureza et al. (2012). Nevertheless, model (2a)–(2d) may contain a huge number of variables because, in general, the number of routes in sets P^l is