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The Traveling Purchaser Problem with time-dependent quantities

E. Angelelli [a], M. Gendreau [b], R. Mansini [c], M. Vindigni [a,*]

[a] Department of Economics and Management, University of Brescia, Italy
[b] CIRRELT and MAGI École Polytechnique de Montréal, Canada
[c] Department of Information Engineering, University of Brescia, Italy

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ABSTRACT

The deterministic Traveling Purchaser Problem looks for a tour visiting a subset of markets in order to satisfy a positive discrete demand for some products at minimum traveling and purchasing costs. In this paper, we assume that the quantities available in the markets for all the products are time-varying decreasing at a constant rate. We propose a compact mixed integer formulation for the problem, and strengthen it with the introduction of connectivity constraints. A new branching strategy and a primal heuristic enforcing the bounding operations have been embedded into a branch-and-cut framework. The branching rule exploits a simple valid inequality and the potential presence of necessary markets. The resulting method outperforms CPLEX 12.6 when used to solve the proposed model. The algorithms have been tested on standard TSPLIB instances, modified to include products and quantities that decrease at different rates of consumption.

1. Introduction

Let us consider a set of products $K := \{1, \dots, n\}$ and a set of markets $M := \{1, \dots, m\}$ plus a depot indexed as 0. For each product $k \in K$ a positive discrete demand d_k is specified. Each product k is offered by a subset of markets $M_k \subseteq M$. For each market $i \in M_k$, q_{ki} units of product k are available at a positive unit price p_{ki} depending on the market i. It is assumed that $\sum_{i \in M_k} q_{ki} \geq d_k$ for all $k \in K$. For each pair i, j of markets and for each market and the depot, a traveling cost c_{ij} is known. The directed Traveling Purchaser Problem (TPP) is defined on a directed graph $G = (V, A)$, where $V := M \cup \{0\}$ is the node set, and $A := \{(i, j)\colon i, j \in V, i \neq j\}$ is the arc set. The problem looks for a simple tour in G starting and ending to the depot and visiting a set of markets so that the demand for each product is satisfied at the minimum routing and purchasing costs.

The problem is known to be NP-hard in the strong sense, and finds relevant applications in routing, scheduling and warehousing contexts [10,11,18]. Several heuristic and exact algorithms have been proposed to solve the problem [26,31] and its several variants [20,29]. More recently, different generalizations to the multi vehicle case with additional resource constraints (capacity, distance traveled) have been proposed [7,33,34].

Frequently, as a static problem the TPP lacks of realism: different purchasers may compete for the same resources, and the quantity supplied in the markets for each product reduces during the day. In this paper, we are interested to study a variant of the TPP where product quantities may decrease over time. The problem is analyzed in a single-day horizon, with fixed product prices and stock replenishment at the beginning of the day. Thus, q_{ki} is the quantity initially available at market $i \in M_k$ for product $k \in K$. Without loss of generality, we also assume that traveling costs satisfy the triangle inequality.

Although in real contexts product consumptions are stochastic processes, we decide to approximate such processes by deterministic ones following the common practice to estimate uncertain quantities by their expected values. Thus, assuming that consumption events follow Poisson processes, we approximate the quantity available over time for product $k \in K$ in market $i \in M$ by a deterministic function depending on a constant positive consumption rate α_{ki}, (see line A in Fig. 1). It is worth noting that the approximation provided by a linear decrease of the quantities over time can safely represent a conservative estimation (a bound value) to the real consumption, when the selected line slope is chosen steep enough (see line B in Fig. 1). One may wonder about the real advantage of introducing a time dependent linear approximation instead of directly applying the static TPP. We are considering real case applications where stock availabilities in the markets are critically low with respect to product demands or situations where the consumption rates are high enough to endanger the demand satisfaction (for a case study we refer the interested readers to

* Corresponding author.
E-mail addresses: enrico.angelelli@unibs.it (E. Angelelli), michel.gendreau@cirrelt.ca (M. Gendreau), renata.mansini@unibs.it (R. Mansini), michele.vindigni@unibs.it (M. Vindigni).

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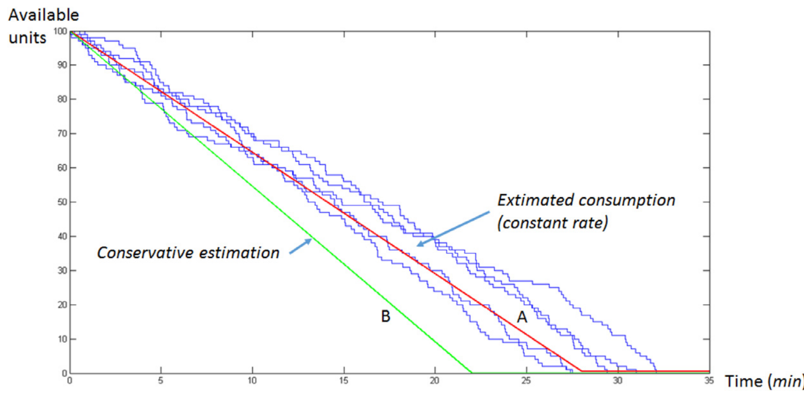

Fig. 1. Linear approximation versus real consumption processes.

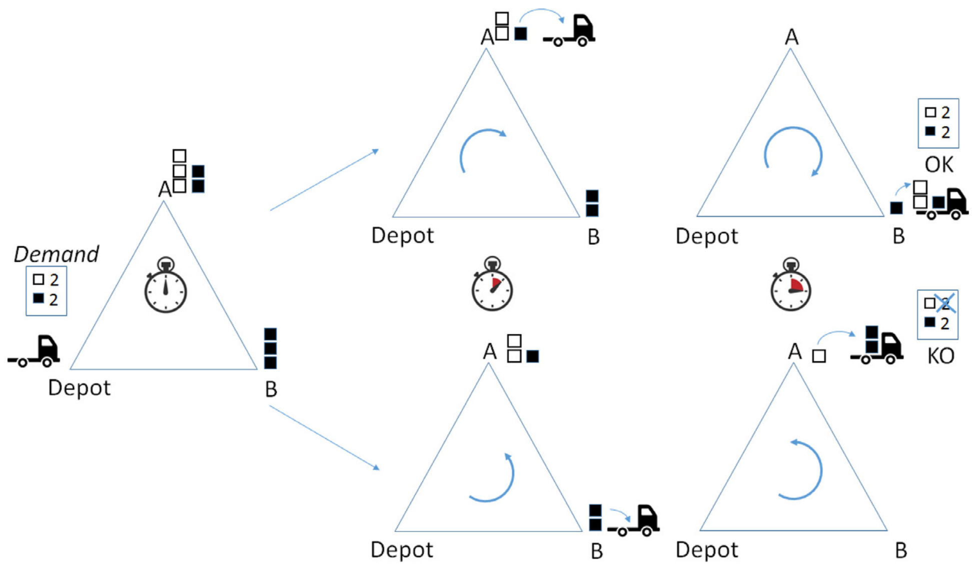

Fig. 2. A wrong markets sequence may result in an infeasible solution: an example.

[4]). In all these cases, feasible solutions are often very difficult to achieve and considering products consumption provides a more accurate way to model reality. Any other approximation of the problem that directly boils down to the static case would not be able to gather the priority constraints that exist among markets and are originated by time-dependent stocks reduction. Thus, the static TPP could only work well in those situations where stocks are far from being quickly exhausted, and consumption rates would probably not affect solutions.

Let t_{ij} be the traveling time required to move from market i to market j, with $i, j \in V$, $i \neq j$. Deciding the sequence of markets to visit in a tour becomes the most critical task of this TPP variant with time-dependent quantities. Indeed, anticipating or postponing a visit to a market may result in an infeasible solution. Fig. 2 shows an instance with two products offered in different quantities in two markets (A and B) that form a triangle with the depot. Each edge has a unitary traveling time. All product quantities decrease at a constant rate equal to one per time unit. The demand for each product is equal to 2 units. Notice that visiting the markets in clockwise order results in a feasible solution, whereas in the reverse order the solution is unfeasible since the quantity available for one of the two products, when the driver reaches market A, is not enough to fully satisfy the demand.

In the literature, different dynamic and stochastic variants of the TPP can be found. In [2,3] the authors analyze a dynamic variant of the problem where the present state of the world is known and the decision maker is informed when some consumption events take place but he has no information on the future. Two classes of heuristic methods have been proposed for the problem; the first one based on myopic criteria to select markets and products and the other one consisting in look-ahead approaches that take some future foresees into account. In [6], the authors study a two-stage stochastic programming formulation of the problem to introduce flexibility to the pure deterministic counterpart. Their policy is a single here-and-now decision related to the selection of the markets and the optimal tour visiting them, whereas recourse decisions are taken in response to each random outcome, related to what and how much to purchase at each market. In [4], the authors analyze the problem by assuming that quantity available in each market for each product decreases over time according to a stochastic process. They cope with the multi-objective nature of the problem through a hierarchical evaluation of the different objectives and introduce three variants of a heuristic approach by using the re-optimization to exploit new information as it becomes available. The proposed approaches are studied under different operating scenarios characterized by the communication technologies at hand and by the level of information available on the state of the world.

The present work provides different contributions with respect to the state-of-the-art literature. First of all the problem is new, and to the best of our knowledge, this is the first time in a routing problem that time-dependent data concern quantities.