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Multi-vehicle prize collecting arc routing for connectivity problem

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ABSTRACT

For effective disaster response, roads should be cleared or repaired to provide accessibility and relief services to the affected people in shortest time. We study an arc routing problem that aims to regain the connectivity of the road network components by clearing a subset of the blocked roads. In this problem, we maximize the total prize gained by reconnecting disconnected network components within a specified time limit. The solution should determine the coordinated routes of each work troop starting at a depot node such that none of the closed roads can be traversed unless their unblocking/clearing procedure is finished. We develop an exact Mixed Integer Program (MIP) and a matheuristic method. The matheuristic solves single vehicle problems sequentially with updated prizes. To obtain an upper bound, we first relax the timing elements in the exact formulation and then solve its relaxed MIP, which decomposes into single vehicle problems, by Lagrangian Relaxation. We show the effectiveness of the proposed methods computationally on both random Euclidean and Istanbul road network data generated with respect to predicted earthquake scenarios.

1. Introduction

Millions of people suffer from natural disasters every year. From the destruction of buildings and infrastructure to the spread of diseases, natural disasters can devastate countries overnight. After a disaster, it is critical to reach the affected areas to provide relief operations such as search and rescue, medical response, aid delivery and establishing temporary shelters. Disasters cause the road networks to be disconnected due to having roads blocked by building debris, fallen lampposts, displaced cars, etc. Moreover, structural damage on the roads also causes blockage. These conditions impede accessibility to casualties, hospitals and supply locations, cause severe handicaps on providing essential resources to the affected people and prevent evacuation activities.

In the immediate disaster response phase, in order to restore the connectivity of the isolated areas, a selected subset of roads should be either rapidly cleared or repaired by road clearing teams consisting of machinery and equipment. We refer to each team as a vehicle from now on. Shortly after information on road conditions is gathered, the vehicles are dispatched from a specific node of the network referred to as the depot.

In this study, we aim to provide an efficient solution method that finds a subset of the closed roads to open and the order in which they should be opened for each vehicle. In order to make the most number of people accessible in short time, we model the problem with the objective of maximizing the collected prize within an imposed time constraint. The prize gained from connecting a network component could be set to the number of people in the isolated component or a priority weight factoring in the locations of facilities such as airports, ports, hospitals and relief supply points within the component. A complexity factor arises since the blocked edges are not traversable unless they are completely unblocked and a vehicle has to wait to enter an edge while it is being unblocked. Hence, node arrival times should be part of the solution. Calculating the arrival time of the vehicles to the nodes complicates the problem since an edge can be traversed multiple times by the same or different vehicles. We call the problem that we study the *Multi-vehicle Prize Collecting Arc Routing for Connectivity Problem* (KPC-ARCP) (here K refers to the number of vehicles). We develop an MIP model to solve the addressed problem. However, the timing related conditions prevent us from finding an optimal solution in realistic-sized instances. Considering that the problem should be solved in short time after the incident takes place, we propose a matheuristic to obtain a near-optimal feasible solution and a relaxation method to derive a tight upper bound to the *KPC-ARCP* which decomposes the problem into single vehicle problems. The heuristic is based on solving K single vehicle MIP models and is easily implementable. Furthermore, the optimality gaps of the heuristic have been found to be zero or very close to zero in the computational study.

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In the next section, we review the related literature and state our contributions. We define the problem in Section 3 and in Section 4 we present the developed mathematical model and in Section 5 we show our solution approach. Section 6 describes data generation and discusses the computational results. We close with concluding remarks in Section 7.

2. Literature review

The problem addressed in this study is in the class of *Arc Routing Problems* (ARP). The most widely known ARP is the *Chinese Postman Problem* (CPP) that seeks for a least-cost closed walk that traverses all the edges [15]. The *Rural Postman Problem* (RPP) is a widely studied extension of the CPP that determines a minimum cost closed walk containing the set of required edges, in addition to other edges that may be traversed [16]. For other variants of ARP, we refer the reader to Corberán and Laporte [13], which is a thorough and up-to-date book on ARP that also provides a categorization of the problems.

The problem we study is built upon the RPP but differs from it in aspects that we will discuss later. The main difference is that in *KPC-ARCP* achieving connectivity is the main concern and the required edges are not known initially and are selected to enable connectivity. Since our objective maximizes the sum of the collected prizes, below we discuss variants of RPP that have a prize collecting or profit maximization objective and other ARPs with synchronization of the vehicles.

While several versions of ARP concentrate on prize collection, node routing problems that address the same type of objectives were studied earlier. Balas [7] introduced the *Prize Collecting Traveling Salesman Problem* (PCTSP), where a salesman gets a prize for each city that he visits and pays a penalty for every city that he does not manage to visit while minimizing the corresponding travel costs and penalties. The *Orienteering Problem* (OP) is another node routing problem that considers a prize collecting type of objective function. In OP, the objective is to collect as much prize as possible in a given tour length limit. The *Team Orienteering Problem* (TOP) is an extension of OP in which multiple teams and their start and end nodes as well as scores (prizes) of the nodes are specified. The teams should leave the start node and traverse several nodes and end their walks in the end node in a given amount of time. The objective is to determine the path for each of the teams from the start point to the end point in order to maximize the total score. For references on TOP see Gavalas et al. [18].

Archetti et al. [5] introduced the arc routing version of TOP which is called *Team Orienteering Arc Routing Problem* (TOARP). Different from TOP in which the prizes can be collected by traversing nodes, in TOARP the prizes should be collected from the arcs of the input graph. Archetti et al. [5] proposed a metaheuristic that finds an optimal solution in almost all the tested instances having up to 27 nodes and 296 arcs.

Several ARPs having a single vehicle and prize collecting or profit maximization objectives have been studied within the last decade. Aráoz et al. [4] introduced the *Privatized Rural Postman Problem* (PRPP), which is also known as the *Prize Collecting Rural Postman Problem* (PCRPP). In PCRPP, a tour starting and ending at the depot, and maximizing the difference between the total prizes from serving required arcs and the total traversing costs is found. The authors developed an integer programming (IP) model with an exponential number of constraints. Aráoz et al. [3] proposed an iterative algorithm together with a heuristic procedure to solve this problem efficiently. The proposed algorithm solves a relaxation of the PCRPP with fewer number of constraints repetitively, while the heuristic generates feasible solutions that provide lower bounds at each iteration.

Feillet et al. [17] study the *Profitable Arc Tour Problem* (PATP) in which a maximum tour length is imposed and there is an upper bound on the number of times a prize can be collected from an arc. Instances with up to 65 nodes were solved using a branch-and-price algorithm. The *Time-Dependent Prize Collecting Arc Routing Problem* (TD-PARP) is another ARP that focuses on collecting prizes within a given tour length limit. Black et al. [10] introduced the TD-PARP in which time dependency refers to travel times that change with the time of the day. Two metaheuristic algorithms, one based on Variable Neighborhood Search and one based on Tabu Search are proposed and tested. The authors solved instances with up to 600 prize arcs using the metaheuristic algorithms.

KPC-ARCP differs from TOARP, PCRPP, PATP and TD-PARP in several ways. For instance, in *KPC-ARCP* connecting components to the supply (depot) node results in gaining prizes which is different from all of these problems. Furthermore, in *KPC-ARCP* a blocked edge can be traversed only after it is opened by a vehicle. This condition, together with the existence of multiple vehicles, complicates the problem since a certain type of synchronization (coordination) of the vehicle routes should be achieved. Although various synchronization issues have been addressed and studied in the literature as discussed below, to the best of our knowledge, this type of synchronization, which involves incurring extra costs only for the first traversal of an edge among multiple vehicles, has been addressed before only in Akbari and Salman [1] that solves a makespan minimization version of *KPC-ARCP*. In Akbari and Salman [1] the goal is to reconnect all the components of a disconnected road network in shortest time. An exact MIP model is given together with a matheuristic approach that includes a local search procedure. In the following we give two examples of ARP with a synchronization requirement.

The *Synchronized Arc Routing Snow Plowing Problem* proposed in Salazar-Aguilar et al. [24] consists of determining a set of routes such that all streets, some of which have multiple lanes, are plowed using fleets of synchronized vehicles. The objective is to minimize the duration of the longest route, namely the makespan. The street segments have one or two directions, and each direction has a number of lanes, typically between one and three. Synchronization occurs since all the lanes of a particular street should be plowed simultaneously. In another study, Salazar-Aguilar et al. [25] introduced the *Synchronized Arc and Node Routing Problem* which is on a directed graph with two sets of arcs: those that must be painted, and those that do not need to be painted. In this problem, a replenishment vehicle to supply paint and a set of homogeneous painting vehicles are used. While the painting is being processed by the painting vehicles, they might run out of paint and the replenishment vehicle should provide them with more paint. The synchronization refers to choosing the appropriate refill nodes such that the painting vehicles and the replenishment vehicle can meet at the same time or without incurring much waiting. Although synchronization has a different meaning in Salazar-Aguilar et al. [24] and [25], it is similar to *KPC-ARCP* in terms of having timing concerns in the mathematical formulation and the heuristic algorithm. While in Salazar-Aguilar et al. [24] all streets should be plowed and in Salazar-Aguilar et al. [25] the set of edges that require the painting are given, in *KPC-ARCP* the required edges to be opened are not given a priori and its part of the decisions. Moreover, while in *KPC-ARCP* a blocked edge can not be traversed unless it is opened, in the discussed problems the vehicles can traverse any arc without providing the services.

Recently, several studies focused on clearing a road network or improving accessibility after a disaster. Some of these studies do not address routing and instead, they focus on selecting the road segments to be cleared or repaired. Duque and Sörensen [14] consider the case where there is a budget constraint, and a number of non-operative roads need to be repaired after a disaster. Weights