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Completion time variance minimisation on two identical parallel processors

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ABSTRACT

The problem of scheduling jobs to minimise completion time variance (CTV) is a well-known problem in scheduling research. CTV is categorized as a non-regular performance measure and its value may decrease by increasing the job completion times. This objective is relevant in situations where providing uniform service to customers is important, and is in-line with just-in-time philosophy. The problem concerned in this paper is to schedule n jobs on *two* identical parallel machines to minimise CTV. We consider the unrestricted version of the problem. The problem is said to be restricted when a machine is not allowed to remain idle when jobs are available for processing. It may be necessary to delay the start of job processing on a machine in order to reduce the completion time deviations. This gives rise to the unrestricted version of the problem. We discuss several properties of an optimal schedule to the problem. In this paper, we develop a lower bound on CTV for a known partial schedule and propose a branch and bound algorithm to solve the problem. Optimal solutions are obtained and results are reported.

1. Introduction

We consider the problem of scheduling n jobs on *two* identical parallel machines with the objective of minimizing the variance of job completion times (CTV). This problem finds application in file handling systems and in just-in-time (JIT) manufacturing. This objective helps in providing uniform service to the customers by bringing the completion times close to each other. In JIT systems both earliness and tardiness are undesirable. The non-linear nature of the objective function leads to larger penalties for larger deviations of job completion times. Examples on non-regular measures can be found in Merten and Muller (1972), Sidney (1977), Kanet (1981b). Graham et al. (1979) introduced the well-known three-field "$\alpha|\beta|\gamma$" notation to represent scheduling problems. The single machine CTV problem, denoted by $(1||CTV)$, and the Mean Squared Deviation (MSD) problem, denoted by $(1||MSD)$, and are two widely studied research problems in scheduling literature. The $(1||MSD)$ problem consists of obtaining a schedule that minimises the mean squared deviation of job completion times about a common due date. Bagchi et al. (1987) showed the equivalence between the $(1||CTV)$ and $(1||MSD)$ problems. They showed that for sufficiently larger due dates both the problems become equivalent to each other.

Merten and Muller (1972) introduced the single machine completion time variance problem where the objective is to minimise the variance of response times to retrieve the data files referenced by the users, especially in on-line systems. They showed that the sequence that minimises the flow-time variance (FTV) is antithetical to the sequence that minimises the waiting time variance (WTV), and thus established the equivalence between FTV and WTV. They also showed that the value of CTV remains the same when the order of the last $(n-1)$ jobs in the sequence is reversed. Schrage (1975) proved that the longest job should be the first in an optimal sequence and conjectured that in an optimal sequence the second and third longest jobs should be in the last and second positions respectively. This conjecture was later proved by Hall and Kubiak (1991). Eilon and Chowdhury (1977) showed that the sequence that minimises the waiting time variance must be V-shaped. A sequence is V-shaped when all jobs preceding (succeeding) the shortest job are in descending (ascending) order of their processing times. Kubiak (1993) proved that the CTV minimisation problem is NP-hard. Numerous heuristic procedures have been proposed by various authors based on the V-shaped property of an optimal sequence: Eilon and Chowdhury (1977), Kanet (1981a), Vani and Raghavachari (1987), Gupta et al. (1990), Manna and Prasad (1999). Gupta et al. (1993) propose a genetic algorithm (GA), Mittenthal et al. (1993) propose a simulated annealing approach, Al-Turki et al. (2001) develop a tabu search based procedure for CTV minimisation on a single machine. Gajpal and Rajendran (2006) proposed an ant-colony optimisation (ACO) algorithm for minimising CTV in a flowshop. De et al. (1992) develop a pseudo-polynomial algorithm to solve this problem. Viswanathkumar and Srinivasan (2003) develop a branch and bound algorithm to obtain an optimal sequence.

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Srinivasan and Srirangacharyulu (2012) develop an improved branch and bound algorithm to solve the single machine CTV problem. However, a little work has been reported on CTV minimisation considering identical parallel machines.

Federgruen and Mosheiov (1996) first addressed the CTV minimisation problem on identical parallel machines, denoted by $(Pm||CTV)$, as a special case of a general cost function. They proposed a lower bound and a simple alternating schedule heuristic for the $(Pm||CTV)$ problem. Two versions of the problem have been studied in literature. The problem is restricted when a machine is not allowed to remain idle when jobs are available for processing. This problem is important when a machine cannot be kept idle, while providing uniform service to the customers. However, in order to obtain a CTV minimising schedule, it may be necessary to delay the start of job processing on a machine. This gives rise to the unrestricted version of the problem. Cai and Cheng (1998) discuss several properties of an optimal solution for the unrestricted version of this problem, denoted by $(Pm|unres|CTV)$. Xu and Ye (2007) showed that the objective function values of $(Pm|unres|CTV)$ and $(Pm|unres|WTV)$ problems are the same at the optimum. They also showed that the optimal solution to one problem can be transformed in to the optimal solution to the other in polynomial time. Chen et al. (2009) considered the restricted version of the problem, denoted by $(Pm|res|CTV)$, and proposed a heuristic procedure. Srirangacharyulu and Srinivasan (2010) discussed both $(Pm|res|CTV)$ and $(Pm|unres|CTV)$ problems and proposed heuristic procedures to solve the problems. Li et al. (2010) addressed the $(Pm|unres|CTV)$ problem and proved several dominant properties of optimal solution. They also proposed a heuristic procedure to solve the problem. Srirangacharyulu and Srinivasan (2011) studied the $(Pm|res|MSD)$ problem. They proposed an algorithm that provide optimal solution to the problem and also proposed a heuristic procedure. However, to the best of our knowledge no algorithm is found to exist in literature to solve $(Pm||CTV)$ problem to optimality.

In this paper we propose a branch and bound algorithm to solve the $(Pm|unres|CTV)$ problem with two identical parallel machines, denoted by $(P2|unres|CTV)$. We discuss several optimality properties, and develop a lower bound on CTV for the $(P2|unres|CTV)$ problem for a given partial schedule. Further we present a branch and bound algorithm and the testing results are reported.

2. Problem description and Notation

The problem is to schedule a set of n independent jobs N={1, 2, …, n} on two identical parallel machines such that the CTV is minimised. Each job has a known processing time and can be processed on any one of the two machines. All jobs are simultaneously available for processing at time zero. Set up times are independent of the job sequence and can be included in job processing times. Job preemption and job splitting are not allowed. The machines are continuously available.

The following notation is used.

N	Set of n independent jobs
M	Set of m identical parallel machines
Λ	Set of all feasible schedules
λ	A feasible schedule
λ^*	An optimal schedule
$CTV(\lambda)$	Completion time variance under λ
$\pi_k(\lambda)$	Ordered set of jobs scheduled on machine k under λ
n_k	Number of jobs scheduled on machine k under λ
p_i	Processing time of job i such that $p_1 \geq p_2 \geq \ldots \geq p_n$
$C_i(\lambda)$	Completion time of job i under λ
$\bar{C}(\lambda)$	Mean completion time under λ
$\overline{C_k}(\lambda)$	Mean completion time of jobs scheduled on machine k under λ
$R_k(\lambda)$	Time when machine k starts to process its first job under λ
σ_{k1}	Ordered set of jobs scheduled on machine k; LPT subsequence
σ_{k2}	Ordered set of jobs scheduled on machine k; SPT subsequence
$\sigma_k=\{\sigma_{k1},\ldots,\sigma_{k2}\}$	Partial schedule on machine k
$\sigma=\{\sigma_1,\sigma_2\}$	Partial schedule, as shown in Fig. 1, with l longest jobs scheduled
n_{k1}	Number of jobs in σ_{k1}
n_{k2}	Number of jobs in σ_{k2}
μ	Complete schedule obtained from σ on completion
D	Sum of processing times of $(n-l)$ shortest jobs
$p^{k1}_{[j]}$	Processing time of the job in position j in σ_{k1}
$p^{k2}_{[j]}$	Processing time of the job in position j in σ_{k2}
$C^{k1}_{[j]}$	Completion time of the job in position j in σ_{k1}
$C^{k2}_{[j]}$	Completion time of the job in position j in σ_{k2}
a_k	Job immediately preceding σ_{k2} in σ
C_{a_k}	Completion time of the job a_k in σ
$\emptyset$	An empty set

The problem is to find λ^* such that

$$CTV(\lambda^*) = \min_{\lambda\in\Lambda} \frac{1}{n}\sum_{k=1}^{2}\sum_{i\in\pi_k(\lambda)}\left(C_i(\lambda)-\overline{C_k}(\lambda)\right)^2 \tag{1}$$

where

$$\overline{C_k}(\lambda) = \frac{1}{|\pi_k(\lambda)|}\sum_{i\in\pi_k(\lambda)} C_i(\lambda) \tag{2}$$

We discuss now some properties of an optimal solution to the $(Pm|unres|CTV)$ problem.

Property 1. (Result 4 in Srirangacharyulu and Srinivasan, 2010). In an optimal schedule, the m longest jobs are scheduled first on each of the m machines.

Property 2. Under $\pi_k(\lambda^*)$ for all $k \in M$, jobs in the second and last positions should be the second and third longest respectively.

Discussion: Since start of job processing on a machine can be delayed, an optimal job sequence on any machine can be obtained as if we were solving a $(1||CTV)$ problem. Hence, the proof follows from Theorem 1 in Hall and Kubiak (1991) and Theorem K in Merten and Muller (1972).

Consider a partial schedule σ as shown in Fig. 1, and let μ be the complete schedule obtained from σ up on completion. Let $d_{(i)}$ denote the ith smallest completion time deviation from $\bar{C}(\mu)$.

Property 3. Given a schedule μ on two machines,

$$\sum_{i=1}^{4} d_{(j+i)} \geq \sum_{i=1}^{j+2} p_{n-i+1} \quad for\ j=0,1,2,\ldots,n-4$$

Discussion: From Lemma 1 in Federgruen and Mosheiov (1996),

$$d_{(j+1)}+d_{(j+2)}+\cdots+d_{(j+2m)} \geq p_n+p_{n-1}+\cdots+p_{n-(j+m-1)},\quad j=0,1,2,\ldots,n-2m$$

Substituting for $m=2$, we get

$$d_{(j+1)}+d_{(j+2)}+d_{(j+3)}+d_{(j+4)} \geq p_n+p_{n-1}+\cdots+p_{n-(j+1)},$$