Computers and Operations Research 81 (2017) 178–191

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Designing and constructing networks under uncertainty in the construction stage: Definition and exact algorithmic approach

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ARTICLE INFO

Article history:
Received 16 June 2015
Revised 31 May 2016
Accepted 22 December 2016
Available online 27 December 2016

Keywords:
Network design
Network construction
Two-stage robust optimization
Exact algorithms

ABSTRACT

The present work proposes a novel Network Optimization problem whose core is to combine both network design and network construction scheduling under uncertainty into a single two-stage robust optimization model. The first-stage decisions correspond to those of a classical network design problem, while the second-stage decisions correspond to those of a network construction scheduling problem (NCS) under uncertainty. The resulting problem, which we will refer to as the Two-Stage Robust Network Design and Construction Problem (2SRNDC), aims at providing a modeling framework in which the design decision not only depends on the design costs (e.g., distances) but also on the corresponding construction plan (e.g., time to provide service to costumers). We provide motivations, mixed integer programming formulations, and an exact algorithm for the 2SRNDC. Experimental results on a large set of instances show the effectiveness of the model in providing robust solutions, and the capability of the proposed algorithm to provide good solutions in reasonable running times.

1. Introduction and motivation

The construction of transportation network systems usually consists of two stages: (i) the strategic network design stage, in which the structure and the composition of the network are defined; and (ii) the construction planning stage, in which a schedule of the construction process in a given time span is established. In other words, in the first stage, the edges of the network to be constructed are *optimally* selected (along with attributes such as capacities), and in the second stage, a schedule for constructing the edges in accordance with the available resources is chosen.

The problems appearing in the first stage fall within the classification of typical Network Design (ND) problems, and a broad body of literature is available (for a recent reference see [22]). Classical examples of ND problems are the minimum spanning tree, MST, problem and the Steiner Tree problem. ND decisions are usually made taking into account data such as geographical distances. Such data is known with complete certainty at the moment of the decision taking, and they are unlikely to change in later stages.

Conversely, the Network Construction Scheduling (NCS) problem of the second stage has been scarcely investigated (see [3]). However, the NCS deserves attention since it defines how and when different elements of the network become available (i.e., how and when the users can access the network) and has a significant impact on the resource requirements of the construction plan. Consequently, NCS affects service revenues as well as construction costs and, therefore, the intrinsic uncertainty of the data (weather conditions, labor efficiency, supplies availability) needs to be taken into account.

Moreover, in order to provide a *reasonable* performance under different situations, including adverse conditions, we believe that network construction must consider the network design and the construction schedule decisions jointly. This approach allows to avoid significant inefficiencies during construction caused by the strict minimization of the network design costs.

Note that the previous setting is not the only situation in which coordinating network design and construction planning provides significant improvements over their separate resolution. For instance, networks subject to frequent disruptions (e.g. disasters) are likely to benefit from considering the tactical decisions (reconstruction of the network after disruptions) during the design process. In such a case, the network should be designed weighting construction costs, possible reconstruction costs, and the cost associated to the deprivation of service until reconstruction. Therefore, the inherent uncertainty associated to disasters must be accounted for.

Two well-known approaches for dealing with uncertainty in optimization are Two-Stage Stochastic Optimization (2SSO) and

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http://dx.doi.org/10.1016/j.cor.2016.12.018
0305-0548/

Robust Optimization (RO). In 2SSO (see [9]) the solutions are built in two stages. In the first stage, a *partial* collection of decisions is defined and completed later on (in the second stage), when the actual data is revealed. Hence, the objective is to minimize the cost of the initial (first-stage) decisions plus the *expected* cost of the recourse (second-stage) decisions. The quality of the solutions provided by this model strongly depends on the accuracy of the random representation of the parameter values (such as probability distributions) that estimate the second-stage expected cost. Nonetheless, sometimes such accuracy is not available, so the use of RO models dealing with *deterministic uncertainty* arises as a suitable alternative (see [7,8,17]). On the one hand, these models do not require assumptions about the distribution of the uncertain input parameters; but on the other hand, they are usually meant for calculating single-stage decisions that are immune (in a sense) to all possible realizations of the parameter values.

A novel alternative that combines RO and 2SSO is Two-stage Robust Optimization (2SRO). As in RO, no stochasticity of the parameters is assumed and, as in 2SSO, decisions are taken in two stages. In this case, the cost of the second-stage decision is computed by looking at the worst-case realization of the data. Therefore, the goal of 2SRO is to find a *robust* first-stage solution that minimizes both the first-stage cost plus the worst-case second-stage cost among all possible data outcomes. 2SRO constitutes a rather generic classification of models; for references on different 2SRO settings we refer the reader to [6,23].

The core of this paper is to combine both network design and network construction under uncertainty into a 2SRO model. The first-stage decisions correspond to those of a classical ND problem, while the second-stage decisions correspond to those of a NCS under uncertainty. The resulting problem, which we will refer to as the Two-Stage Robust Network Design and Construction Problem (2SRNDC), aims at providing a modeling framework in which the design decision not only depends on the design costs (e.g. distances), but also on the corresponding construction plan. In other words, an optimal first-stage designed network, say N^*, not only ensures efficiency with respect to the design cost but it also ensures that the induced second-stage construction cost will be, in the worst case, as cheap as possible.

Among the possible 2SRNDC formulations, this paper focuses on the case in which a spanning tree has to be constructed and the scheduling decisions try to minimize the time before users have access to the network. The scheduling objective is represented as the sum of connection times to a central node (e.g. depot or service provider). The resulting problem will be referred to as the Two-stage Robust Spanning Tree Sum of Connection Times problem (2SRSTSCT).

1.1. Our contribution

The contribution of this paper is threefold. First, we contribute to the new field of problems dealing with network construction scheduling by providing a model that combines both stages of decision (design and construction). Second, we analyze the uncertainty introduced by construction, departing from the classical approach that associates uncertainty to network design. This is a novel approach as the source of uncertainty introduces a secondary optimization problem instead of splitting the decision in two decision stages. And third, we provide an algorithmic framework to tackle this new type of problems. Such a framework has been tailored for a particular version of the problem, but this exposition stresses that the proposed algorithm can be adapted to other variants with a different first-stage network design problem or a different second-stage scheduling problem. The version of the problem considered in this work corresponds to essential versions of both the first and the second stage. The first stage selects a spanning tree. The second stage orders the construction of the edges of the spanning tree while minimizing the average connection time from a central node (the depot or service provider) to the nodes of the network (customers). The formulation of the scheduling problem has been shown to resemble a single machine scheduling problem with sum of completion times objective (see [3]).

1.2. Previous work

We now provide a brief review of recent publications dealing with ND combined with multi-period scheduling decisions and with ND combined with 2SRO.

Network construction scheduling problems. The model studied in [21] corresponds to an infrastructure restoring problem in which the goal is to locate a set of work groups to install additional arcs into a disrupted network, with the aim of enabling the flow of a commodity from supply to demand nodes. A Mixed Integer Programming (MIP) formulation is proposed, reduction techniques are devised, heuristic procedures are designed, and extensive numerical results on a set of realistic instances are reported.

In [5,13,15] the authors study the problem of *completing* a network (mainly by adding new edges) in T periods ensuring that in each period $t \in \{1, \dots, T\}$ an optimally designed sub-network, $N^*(t)$, is obtained from the network completed up to that period. In [13], a minimum spanning tree must be solved at each period; in [5], a shortest path has to be found at every t. Finally, in [15], $N^*(t)$ corresponds to an allocation of flow that ensures a maximum flow between a fixed pair of nodes. Formulations, heuristic and approximation algorithms, reduction techniques, and computational results are reported.

A more recent general framework for integrating network design and scheduling decisions is proposed in [20]. This new approach generalizes the model previously proposed in [21]. In this case, at each period, the performance of the network expansion strategy is evaluated with respect to different network models: maximum flow, minimum-cost flow, shortest path, and spanning tree. Moreover, two types of objective functions are considered: a *cumulative* objective (which aims at optimizing the weighted network performance over the time span), and a *threshold* objective (which aims at minimizing the time needed to *reach* the predefined performance value).

Another alternative to combine network design and scheduling decisions corresponds to the Network Construction Scheduling Problem (NCS) defined in [3]. In the NCS, the edges of a network need to be constructed in order to provide connectivity between a node (the depot), and the remaining nodes (the customers). The objective is to obtain a construction order for the selected edges that optimizes a metric associated to the time required to connect a given node (the service provider) to every other node of the network.

This formulation allows the use of different scheduling objectives, like the minimization of the weighted or unweighted sum of completion times (equivalent to minimizing average connection times) [3], the minimization of the maximum lateness (equivalent to minimizing the maximum delay with respect to the due date of each node), or the minimization of the number of tardy nodes (equivalent to minimizing the total number of unfulfilled contractual dates) [4]. These objectives describe problems in which the connection time between a source of service (the depot) and the remaining nodes (the customers) is critical.

In this work the scheduling decision falls within the NCS framework, and we consider the minimization of the average time required to connect the nodes, which is equivalent to the unweighted sum of completion times. This objective constitutes a valid construction performance when no additional information is available