Value of agreement in decision analysis: Concept, measures and application

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ABSTRACT

In multi-criteria decision analysis workshops, participants often appraise the options individually before discussing the scoring as a group. The individual appraisals lead to score ranges within which the group then seeks the necessary agreement to identify their preferred option. Preference programming enables some options to be identified as dominated even before the group agrees on a precise scoring for them.

Workshop participants usually face time pressure to make a decision. Decision support can be provided by flagging options for which further agreement on their scores seems particularly valuable. By valuable, we mean the opportunity to identify other options as dominated (using preference programming) without having their precise scores agreed beforehand. The present paper quantifies this Value of Agreement and extends the concept to portfolio decision analysis and criterion weights. The new concept is validated through a case study in recruitment.

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1. Introduction

Multi-criteria decision analysis (MCDA) often assesses options with an additive independent value function [31]. The performance of each option $i \in I$ is valued on each criterion $j \in J$ along a 0–100 scale to obtain the value scores v_{ij} (see Table 1 for symbols). While the value score for tangible criteria can often be derived from well-defined, marginal value functions, which map attribute levels to value scores (e.g. frequency of a service, horsepower of a machine, traffic noise), many intangible criteria require highly subjective judgements to assign value scores to options (e.g. impact of constructions on scenery, level of expertise of organisation, comfort of vehicle) [28]. The criteria are weighted against each other, leading to the relative importance judgements w_j with $\sum_j w_j = 1$. The decision model recommends the option with the greatest score $V_i = \sum_j w_j v_{ij}$.

Real-world problems are rarely that simplistic, and extensions to the model are therefore required. The present paper addresses MCDA problems in which the score preferences are initially incomplete; i.e. the criterion scores can take any number in the range $[\underline{v}_{ij}, \bar{v}_{ij}]$. The options' value scores hence span from $\underline{V}_i = \sum_j w_j \underline{v}_{ij}$ to $\bar{V}_i = \sum_j w_j \bar{v}_{ij}$. If there is one option that dominates all others, it is called the robust option [63].

Such incomplete preference information [25,38,72] is often encountered by decision-making groups appraising options against intangible selection criteria. In multi-criteria group decision making, participants frequently appraise the options individually before seeking agreement on the scores. The individual precise criterion scores v_{ijk} by participants $k \in K$ lead to criterion score ranges from $\underline{v}_{ij} = \min_k\{v_{ijk}\}$ to $\bar{v}_{ij} = \max_k\{v_{ijk}\}$, within which the group looks for the necessary agreement to identify the group's robust option. 'Aggregation' and 'consensus' are the two principle methods to reach this necessary agreement. Using the aggregation method, an influence weight is assigned to each participant (e.g. [17,37,66]). A precise group score for all decision options can be automatically calculated by weighting the individual scores, which immediately clarifies the group's preferred option according to the participants' influence. Using the consensus method, the group engages in a discussion from which the necessary consensus on the scores needs to emerge with mutual agreement (e.g. [16,45,49,57,73,84]). The aggregation method stresses the positional power of each decision maker, while the consensus method emphasises mutual learning from each other's insights in the decision problem. When using the consensus method, time pressure may make it challenging for the group to come to a full agreement on all scores [36,87]. In this paper, we develop a new concept, which helps the consensus method in multi-criteria group decision-making to be more time-effective.

When thoroughly reviewing the individual appraisals $V_{ik} = \sum_j w_j v_{ijk}$ and the resulting score ranges from $\underline{V}_i = \min_k\{V_{ik}\}$ to $\bar{V}_i = \max_k\{V_{ik}\}$, a skilled facilitator might be able to roughly guess

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Table 1
Symbols.

B	Budget
c_i	Cost of option i
ε	Tolerance value
i/I	Index/set of options
i^*	Robust option
j/J	Index/set of criteria
k/K	Index/set of assessors (workshop participants)
θ	Parameter for social judgement scheme
$\varphi/\hat{\varphi}$	Value score of the robust option in the MCDA case; borderline value-for-money ratio in the PDA case; $\hat{\varphi}$ is the heuristic choice for φ
r_i/r_i^*	Value-for-money ratio of option i; r_i^* is the prediction for r_i
S_w	Set of extreme points of the convex hull of feasible weight combinations
$v_{ij}/v_{ij}^*/v_{ij}^a/v_{ij}^p$	Value score of option i for criterion j; v_{ij}^*, v_{ij}^a and v_{ij}^p are, respectively, the predicted, the actually agreed and the proposed value for v_{ij}
$\underline{v}_{ij}/\bar{v}_{ij}$	Lower/upper bound for v_{ij}
$\underline{v}_{i'j}^{\#i}/\bar{v}_{i'j}^{\#i}$	Lower/upper bound for $v_{i'j}$ after assuming that v_{ij} takes its predicted value v_{ij}^*
v_{ijk}	Value for v_{ij} assigned by assessor k
$V_i/V_i^*/V_i^a$	Overall value score of option i; V_i^* and V_i^a are, respectively, the predicted and the actually agreed value for V_i
$\underline{V}_i/\bar{V}_i$	Lower/upper bound for V_i
$\underline{V}_{i'}^{\#i}/\bar{V}_{i'}^{\#i}$	Lower/upper bound for $V_{i'}$ after assuming that V_i takes its predicted value V_i^*
$\underline{V}_i^{\#j}/\bar{V}_i^{\#j}$	Lower/upper bound for V_i after assuming that w_j takes its predicted value w_j^*
V_{ik}	Value for V_i assigned by assessor k
$w_j/w_j^*/w_j^a$	0–1 normalised weight of criterion j; w_j^* and w_j^a are, respectively, the predicted and the actually agreed value for w_j. A tilde $\sim$ on top indicates that the weight is not normalised.
w_{jk}	0–1 normalised value for w_j assigned by assessor k. A tilde $\sim$ on top indicates that the weight is not normalised.
$\underline{w}_j/\bar{w}_j$	0–1 normalised lower/upper bound for w_j. A tilde $\sim$ on top indicates that the weight is not normalised.
z_{ijk}	Centrality position of group member k for the assessment of score v_{ij} according to the social judgement scheme

which options are definitely dominated, which options are likely to be dominated after further agreement on some other options is sought, which ones have a totally unclear fate and which ones have a good chance of becoming the robust one. The present paper attempts to replace this intuitive guessing with a quantitative concept we call Value of Agreement. Options with a high Value of Agreement are more attractive to a group discussion as knowing their agreed value score may allow eliminating other options as dominated without the need to seek agreement on their scores beforehand; thus, finding the robust option will require less effort. Altogether, the Value of Agreement estimates the impact of eliciting additional preference information on the time requirements of a decision analysis workshop.

Designing workshops to eliminate options quickly has hitherto been addressed only by very few publications. Hämäläinen and Pöyhönen [23] and Salo and Hämäläinen [68] analysed problems where only the weight information was incomplete. They suggested that the group should first seek agreement on the criteria with large weight ranges. Mustajoki and Hämäläinen [46,47] provided decision aid to wisely choose the next swap in the MCDA method even swaps. Mustajoki et al. [48] studied which criterion should be used as the reference criterion for the weight elicitation to harness preference programming effectively. Liesiö et al. [42,43] briefly examined the problem for portfolio decision analysis but only offered limited guidance for an option elicitation order.[1]

The paper proceeds as follows: Section 2 briefly discusses normative preference programming techniques to identify definitely dominated options. Section 3 explains descriptive approaches developed by psychologists to predict on which precise scores v_{ij}^* the group may finally agree. Section 4 pulls together the normative and descriptive decision-making perspectives from the previous two sections and develops a prescriptive measure for the Value of Agreement on option scores in MCDA. Section 5 modifies this measure for multi-criteria portfolio decision analysis (PDA)—an important extension of MCDA. Section 6 adapts the Value of

Agreement to weights. Section 7 demonstrates the effectiveness of the proposed measures with a computer simulation. Section 8 reports on the application of the Value of Agreement concept to a PDA-based recruitment selection problem using computer software. Section 9 concludes with a research outlook.

2. Preference programming

Preference programming encompasses a set of techniques to eliminate definitely dominated options and ultimately identify the robust option given the incomplete information about scores and weights [2,34,61,67]. The expressed incomplete preferences are typically range-based or ordinal-based (e.g. [38,42,61]). Preference programming is a normative approach that, in its original form, captures the incomplete information in a set of linear constraints. Non-dominated options can be easily identified by examining the extreme points of the resulting convex hull (e.g. [25,42,88]) or by solving mathematical programmes (e.g. [3,39,68]).

The *range-based strict dominance rule* (e.g. [25])

$$i_1 \succ i_2 \Leftrightarrow \underline{V}_{i_1} \geq \bar{V}_{i_2} \text{ and } \bar{V}_{i_1} > \bar{V}_{i_2}$$

is a logic based on preference ranges which can be applied in a basic MCDA problem where just the score information is incomplete. For the robust option i^*, the strict preference $i^* \succ i$ must hold for all $i \in I\backslash\{i^*\}$.[2]

An *ordinal-based weak dominance rule* can be constructed by considering participants' implicit rankings when assigning scores to options. Option i_1 weakly dominates option i_2 on criterion j if all participants k believe that $v_{i_1 jk} \geq v_{i_2 jk}$. In this case, the linear constraint $v_{i_1 j} \geq v_{i_2 j}$ can be added to the preference programme. Option i_1 weakly dominates options i_2 overall if all par-

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[1] Following their argument, options with a core index of around 0.5 should probably be examined first.

[2] As an alternative to Hazen's strict dominance rule, one may apply a quasi-dominance rule instead [16]. In this case, an option i^* is defined as robust if no other option $i \in I\backslash\{i^*\}$ with an upper bound $\bar{V}_i$ exceeding the lower bound $\underline{V}_{i^*}$ of option i^* by more than a given tolerance value ε exists. Quasi-dominance allows taking into account that the maximum loss of value the group could suffer when choosing option i^* may be too small to justify letting the decision making group convene for longer [70]. Quasi-dominance would add more complexity to the decision model and therefore is not discussed further in this paper.