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Abstract

In this article, we consider bi-dimensional knapsack problems with a soft constraint, i.e., a constraint for which the right-hand side is not precisely fixed or uncertain. We reformulate these problems as bi-objective knapsack problems, where the soft constraint is relaxed and interpreted as an additional objective function. In this way, a sensitivity analysis for the bi-dimensional knapsack problem can be performed: The trade-off between constraint satisfaction, on the one hand, and the original objective value, on the other hand, can be analyzed. It is shown that a dynamic programming based solution approach for the bi-objective knapsack problem can be adapted in such a way that a representation of the nondominated set is obtained at moderate extra cost. In this context, we are particularly interested in representations of that part of the nondominated set that is in a certain sense close to the constrained optimum in the objective space. We discuss strategies for bound computations and for handling negative cost coefficients, which occur through the transformation. Numerical results comparing the bi-dimensional and bi-objective approaches are presented.

Keywords: bi-dimensional knapsack problem, bi-objective knapsack problem, sensitivity analysis, soft constraints, dynamic programming

1. Introduction

Given a finite set of items with positive profits, weights, and a finite capacity, the 0–1-knapsack problem decides whether or not to include items, where each item can be included at most once. The goal is to maximize the overall profit of the included items under the constraint that the overall weight does not exceed the given capacity.

The knapsack problem (KP) is a classical problem in combinatorial optimization. It has applications in project selection, capital budgeting, and many others, and it appears as a frequent subproblem in more complex situations such as, for example, network design. Martello and Toth (1990) and Kellerer

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