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A novel discretization scheme for the close enough traveling salesman problem

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ABSTRACT

This paper addresses a variant of the Euclidean traveling salesman problem in which the traveler visits a node if it passes through the neighborhood set of that node. The problem is known as the close-enough traveling salesman problem. We introduce a new effective discretization scheme that allows us to compute both a lower and an upper bound for the optimal solution. Moreover, we apply a graph reduction algorithm that significantly reduces the problem size and speeds up computation of the bounds. We evaluate the effectiveness and the performance of our approach on several benchmark instances. The computational results show that our algorithm is faster than the other algorithms available in the literature and that the bounds it provides are almost always more accurate.

1. Introduction

This paper concerns the close-enough variant CETSP of the classic traveling salesman problem (TSP). Given a set of target points in any Euclidean space, the TSP consists in determining a minimum length tour that starts and ends at a ”depot” while visiting each target point exactly once. In CETSP to each target point v is associated a neighborhood, that is a compact region of the space containing v. In fact CETSP consist in finding the shortest tour that starts and ends at the depot and intersects each neighborhood once. Usually, there are no constraints on the shape of the neighborhood, on the other hand the disc shape is the mostly adopted one, thus we keep this assumption in our paper.

The CETSP has a number of practical applications. For instance, let us consider a district where a certain number of radio frequency identification (RFID) readers are located to record electricity or water or gas consumption. Each meter reader plays the role of a target point and its information can be relayed within a fixed range r. Consequently, the neighborhood is defined as a disc of radius r centered at each target point. The reading process of the RFID reader meters can be performed by flying a drone within the neighborhood of each target point, speeding up the classic door-to-door reading. Other applications of CETSP arise in the robot monitoring of wireless sensor networks [13] and in the context of Unmanned Aerial Vehicles for aerial forest fire detection or military surveillance.

The CETSP was introduced by Gulczynski et al. [7] and the authors proposed six heuristics to solve the problem under the assumption that all neighborhoods were discs of the same radius. For the same problem, Dong et al. [4] introduced two heuristics based on the concept of supernodes. A supernode S is a set of points of the plane such that for each target point v there exists at least one point in S whose distance from v is at most r, the radius of the disc. Supernodes are generated by using convex hull and clustering techniques. A mixed integer nonlinear programming formulation of CETSP was provided too, but it was not specifically used in algorithm design. Mennell et al. [10,11] proposed another heuristic based on the intersection of the neighborhoods, named Steiner zone. Yuan et al. [13] developed an effective evolutionary approach which was able to find the shortest tour on all the benchmark instances, although with large computation time. Other heuristics were proposed by Shuttleworth et al. [12] to solve the CETSP over a street network for the specific RFID meter reader application described above. Finally, some special cases of CETSP were solved by polynomial-time approximation algorithms introduced by Arkin and Hassin [1], Mata and Mitchell [9] and Dumitrescu and Mitchell [5].

In this paper we introduce an approach to compute upper and lower bounds for the CETSP problem by discretizing the solution space and solving on the resulting graph the classic Generalized TSP problem. Starting from the discretization approaches already

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proposed in the literature, we introduce a new effective discretization scheme which provides better bounds, thanks to a novel adaptive approach to select appropriately the number of discretization points to be used for each neighborhood. Finally, we apply a graph reduction algorithm that significantly reduces the size of the Generalized TSP problem to solve, allowing us to increase the number of discretization points without penalizing the performance of our approach in terms of CPU time. The computational results, carried out on benchmark instances, reveal that our approach outperforms the ones proposed in [2].

The remainder of the paper is organized as follows. Section 2 introduces the definitions and the notations that are used throughout the paper. Sections 3 and 5 present our discretization scheme and our graph reduction algorithm, respectively. The mixed-integer programming model (MIP) is described in Section 6 and it is followed by the computational results in Section 7. Finally, conclusions are presented in Section 8.

2. Definitions and notation

Let N be a set of points in a two-dimensional plane, with $|N| = n$, and let p_0 be the depot point. We will refer to the elements in N as the *target points*. To each target point v is associated a sphere C_v with center v and radius r_v (Fig. 1(a)) which will be referred to as the *neighborhood* $N(v)$ of v. W.l.o.g., we suppose that $p_0 \notin N(v)\ \forall\ v \in N$. The CETSP consists in finding a shortest tour T^* that starts from the depot p_0, intersects every neighborhood $N(v)$ (in any order), and ends in p_0. The points of any tour T where a direction change occurs are the *turn points* and any tour can be uniquely identified through its turn points. For instance, in Fig. 1 (b) it is shown a feasible tour T for the CETSP which is identified by the turn points p_1, p_2 and p_3. Given a couple of turn points p_i and p_j, the length of the edge (p_i, p_j) is given by the Euclidean distance between p_i and p_j. The total cost of a tour T is denoted by $w(T)$ and it is equal to the sum of the edge lengths in T.

Given the neighborhood $N(v)$ depicted in Fig. 1(a), let d_i and d_j be two points of the boundary of C_v. We denote by $\overline{d_i, d_j}$ the *chord* between these two points and by $\widehat{d_i, d_j}$ the *circular arc* from d_i to d_j in the clockwise direction. For any additional definition and notation on the graphs we refer to [3].

3. The perimetral discretization scheme

Since each neighborhood $N(v)$, $v \in N$, contains an infinite number of turn points, then the number of feasible tours for CETSP is infinite as well. However, a finite number of turn points occur for any feasible tour. For this reason, we can associate to each feasible solution a discrete set of points. More in detail, each neighborhood $N(v)$ is discretized by using a fixed number k of discretization points. We denote by $\hat{N}(v)$ such set of points. Consequently, a graph $G = (V, E)$, where $V = \bigcup_{v \in N} \hat{N}(v)$ and $E = \{(x, y): x \in N(u), y \in N(v), u \neq v\}$, is build. It is easy to see that the weight of any tour $\hat{T}$, that starts and ends at the depot and that visits exactly one discretization point in each neighborhood, is an upper bound to $w(T^*)$. From now on, we will denote by T and $\hat{T}$ the feasible tours of the CETSP computed by using the points of $N(v)$ and of $\hat{N}(v)$, $v \in N$, respectively.

In order to have an upper bound of $w(T^*)$ as tight as possible, we compute the shortest tour $\hat{T}^*$ in G, namely we solve the *Generalized TSP* problem (GTSP) on G. The quality of the bound $\hat{T}^*$ for $w(T^*)$ heavily depends on the number of points used to carry out the discretization and on their placement in each neighborhood. Obviously greater the number of discretization points tighter will be $w(\hat{T}^*)$, but, on the other hand, greater will be the size of G with increasing computational cost to calculate $\hat{T}^*$. For this reason it is necessary to find an appropriate trade-off between the quality of the upper bound and the time spent to compute it. Moreover, it is crucial to use a discretization scheme that minimizes the discretization error carried out in each neighborhood due to the use of discretization points. More in details, let us consider the example in Fig. 2 where $N = \{v_1, v_2, v_3\}$ and T^* is the optimal tour for the CETSP, identified by the turn points p_1, p_2, p_3 and the depot p_0. Note that the turn points of T^* are always on the boundary of the spheres (see Proposition 1 in the sequel). Each neighborhood is discretized by using only k=2 discretization points placed on the corresponding circumference.

Let us build now the *walk* $Q = \{p_0, p_1, d_1, p_1, p_2, d_2, p_2, p_3, p_0\}$. In practice Q is built by following the edges of T^* and, for each turn point $p_i \in C_{v_i}$, the closest discretization point $d_i \in \hat{N}(v_i)$ is detected and the chord $\overline{p_i, d_i}$ is crossed twice. We define the *discretization error* $\xi(v_i)$ as two times the length of $\overline{p_i, d_i}$. Thus $\xi(v_i)$ represents the error in $\hat{N}(v_i)$ with respect to T^*, due to the choice of the discretization points. It is easy to see that:

$$w(Q) = w(T^*) + \sum_{v \in N} \xi(v) \tag{1}$$

Since Q starts and ends at the depot p_0 and visits one

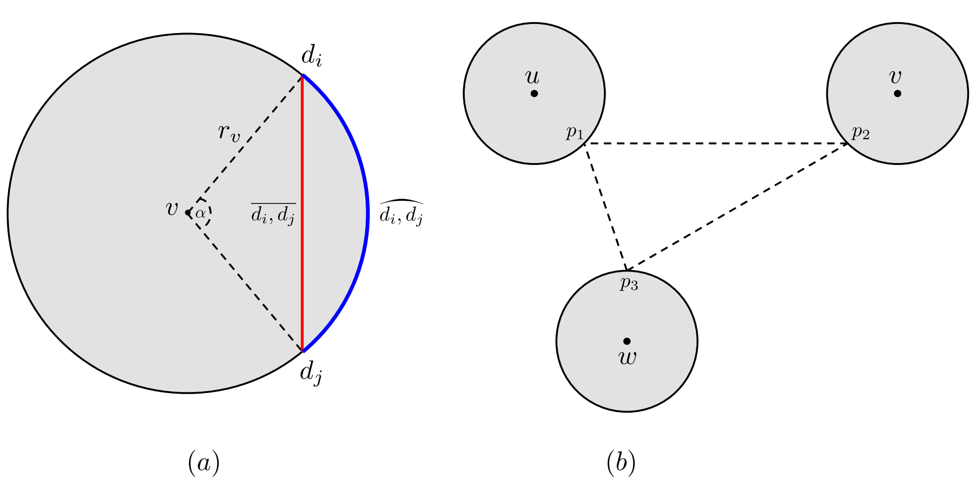

Fig. 1. (a) Neighborhood $N(v)$ of the target point v. (b) A feasible tour for the CETSP problem.