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PII: S0305-0548(16)30208-8
DOI: http://dx.doi.org/10.1016/j.cor.2016.08.013
Reference: CAOR4068

To appear in: *Computers and Operation Research*

Received date: 29 January 2016
Revised date: 19 June 2016
Accepted date: 24 August 2016

Cite this article as: B. Golany, N. Goldberg and U.G. Rothblum, A Two-Resource Allocation Algorithm with an Application to Large-Scale Zero-Sum Defensive Games, *Computers and Operation Research* http://dx.doi.org/10.1016/j.cor.2016.08.013

A Two-Resource Allocation Algorithm with an Application to Large-Scale Zero-Sum Defensive Games

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August 24, 2016

Abstract

This paper investigates efficient computation schemes for allocating two defensive resources to multiple sites to protect against possible attacks by an adversary. The availability of the two resources is constrained and the effectiveness of each may vary over the sites. The problem is formulated as a two-person zero-sum game with particular piecewise linear utility functions**: the expected damage to a site that is attacked linearly decreases in the allocated resource amounts up to a point that a site is fully protected. The utility of the attacker, equivalently the defender's disutility, is the total expected damage over all sites.** A fast algorithm is devised for computing the game's Nash equilibria; it is shown to be more efficient in practice than both general purpose linear programming solvers and a specialized method developed in the mid 1980s. To develop the algorithm, optimal solution properties are explored.

Key words: Resource allocation, multiple resources, resource substitution, Nash equilibria, large scale optimization

1 Introduction

The problem of allocating multiple scarce resources to optimize an objective function, in particular a linear one, is one of the classic problems in Operations Research. These problems can be traced back to the foundation of linear programming (LP); see for example early work of Dantzig, Hitchcock, Kantorovich and Koopmans among others [9], the surveys of Luss [20] and Katoh and Ibaraki [15]. Also, Luss [19] and references therein provide a perspective on the extensive study of this problem. In a competitive (game theoretic) setting some parameters of the allocating agent's utility function are determined by an adversary.

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‡Posthumous.