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A planar single-facility competitive location and design problem under the multi-deterministic choice rule

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ABSTRACT

A new customer choice rule, which may model in some cases the actual patronising behaviour of customers towards the facilities closer to reality than other existing rules, is proposed. According to the new rule, customers split their demand among the firms in the market by patronising only one facility from each firm, the one with the highest utility, and the demand is split among those facilities proportionally to their attraction. The influence of the choice rule in the location of facilities is investigated. In particular, a new continuous competitive single-facility location and design problem using this new rule is proposed. Both exact and heuristic methods are proposed to solve it. A comparison with the classical proportional (or Huff) choice rule when solving the location model reveals that both the location and the quality of the new facility to be located may be quite different depending on the patronising behaviour of customers. Most importantly, the profit that the locating chain may lose if a wrong choice is made can be quite high in some instances.

1. Introduction and notation

The estimation of the market share that can be captured by a facility in a competitive environment where there exist other facilities offering the same product is a topic of major concern for managers, as the survival of a facility depends on the revenues it can obtain, and those revenues largely depend on the market share. Where to locate a facility is a strategic decision which cannot be easily altered as the location of a facility usually requires a massive investment. But how do we choose the right location for a new facility?

Competitive location problems concerning optimally placing facilities in a competitive environment have been widely developed for a number of contextual applications in the traditional retail sector, see for instance the survey papers of Eiselt and Laporte [10], Eiselt et al. [11] and Plastria [29] and the references therein. They vary in the ingredients which form the model. For instance, the *location space* may be the plane, a network or a discrete set. We may want to locate just one or more than one new facility. The competition may be *static*, which means that the competitors are already in the market and the owner of the new facility knows their characteristics, or *with foresight*, in which the competitors are not in the market yet but they will be soon after the new facility enters. Demand is usually supposed to be concentrated in a discrete set of points, called *demand points*, and it can be either *inelastic* or *elastic*, depending on whether the goods are essential or inessential.

It is also necessary to specify what the *attraction (or utility) function* of a customer towards a given facility is. Usually, the attraction function depends on the distance between the customer and the facility, as well as on other characteristics of the facility which determine its *quality*.

The *patronising behaviour* of the customers must also be taken into account, since the *market share* captured by the facilities depends on it. This is the topic this paper is devoted to. For instance, it is not uncommon to see in the literature papers where consumers shop at the closest store supplying a specific product or service. But, does this assumption reflect consumer behaviour? It seems more realistic to admit that consumers do not merely consider distance when choosing retail outlets. Also, consumers may patronise more than one facility to satisfy their demand. Consumer choice behaviour literature studies the key variables that a customer takes into account to patronise one or

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another facility, and how these variables interact.

A common classification of the consumer choice behaviour states that this can be done in three groups [5]:

- The first one includes models that rely on some "normative assumption" regarding consumer travel behaviour. This hypothesis is too simple and is useful only in a limited number of applications. The classic example is the so-called *deterministic rule*, which says that "consumers patronise the nearest outlet that provides the required goods or service". This hypothesis has not found much empirical support, except in areas where shopping opportunities are limited and transportation is difficult.
- The second group uses information revealed by past behaviour to understand the dynamics of retail competition and how consumers choose among alternative shopping opportunities. Huff [24] was the first one to use the revealed preference approach to study retail store choice. The Huff probability formulation, known as the *probabilistic rule*, uses distance (or travel time) from consumer zones to retail centres and the size of retail centres as inputs to find the probability of consumers shopping at a given retail outlet.
- The third group of models estimates the consumer utility function from simulated choice data using information integration, conjoint or logit techniques. Instead of observing past choices, these methods use consumer evaluations of hypothetical store descriptions to calibrate the utility function. The best representative model of this group is the one developed by Ghosh and Craig [16] based on game theory.

Clarkon et al. [3] have pointed out that firms prefer the revealed preference approach to model consumer store-choice behaviour. This approach is preferred to normative models since it more faithfully reflects real consumer behaviour, and to the direct utility approach because it is simpler since it uses surveys and linear regression instead of conjoint, logit techniques or game theory. We follow the revealed preference approach in this paper.

The two customer choice rules commonly used in literature are the following:

Deterministic (or binary) rule: it assumes that the full demand of a customer is satisfied by *only one* centre, the one to which he/she is attracted most, disregarding all other facilities which are less attractive, even those whose difference in attraction is very small.

Probabilistic rule: it assumes that a customer splits his/her demand probabilistically over *all* facilities in the market proportionally to his/her attraction to each facility.

Hotelling [22] was the first to propose the deterministic choice rule for a simple model on a line. That is why competitive location models using this rule are also referred to as Hotelling models. The first two papers that introduced location models in a more general space assuming that customers patronise the closest facility were Drezner [9] in the plane and Hakimi [17] on a network.

Huff [23,24] described the gravity model suggested by Reilly [36], although he did not investigate any location problem. The first paper that considered the location problem based on the Huff rule was Drezner [6]. Later on, Fernández et al. [14] and Aboolian et al. [1] introduced the design as an additional variable of the model, although an earlier version of location and design was already introduced in Drezner [7].

The aim of this paper is twofold. First, we present a new choice rule, named multi-deterministic choice rule, which may, in some cases, model the patronising behaviour of customers closer to reality than other existing rules in many practical applications. In particular, we introduce a new single-facility location and design problem on the plane which considers this rule. Second, we investigate up to what extent the selection of the choice rule may affect the location decisions of a firm that wants to expand its presence in a given geographical region by opening new facilities. In particular, we will compare the outputs provided by models using the probabilistic and the multi-deterministic rules on the same input data sets.

In the rest of the paper, in order to fix ideas, we assume the following scenario (notice, however, that the main conclusion from the paper, i.e., that the selection of the right customer choice rule is a critical issue for the location decisions of a firm that wants to set up new facilities, remains valid for other competitive location models as well): A chain wants to locate a new single facility in a given area of the plane, where there already exist other facilities in the vicinity offering the same goods or product. Some of those facilities may belong to the locating chain. The demand is supposed to be fixed and concentrated at given demand points, whose locations and buying powers are known, as well as the location and quality of the existing facilities. The attraction of a demand point towards a facility is modelled multiplicatively as quality divided by perceived distance. This generalises the law of retail gravitation of Reilly [36], who considered the perceived distance to be the squared distance. Quality was first estimated as store surface by Huff [24], and later several other store characteristics were incorporated by Jain and Mahajan [25] and Nakanishi and Cooper [28]. For details see Drezner and Eiselt [8].

The objective is to maximise the *profit* obtained by the chain after the location of the new facility, to be understood as the income due to the market share captured by the chain minus its operational costs. Both the location and the quality of the new facility are to be found.

In order to give a mathematical formulation of location models using the different customer choice rules, the following notation will be used:

Indices

i	index of demand points, $i = 1, \ldots, i_{\max}$.
c	index of competing chains, $c = 1, \ldots, c_{\max}$ (chain c=1 is the locating chain).
j	index of existing facilities, $j = 1, \ldots, j_{\max}$ (we assume that from $j = j^1_{\min}(=1)$ to $j^1_{\max}$ the facilities belong to chain c=1 ($j^1_{\max} < j_{\max}$); from $j = j^2_{\min}(= j^1_{\max} + 1)$ to $j^2_{\max}$ belong to chain c=2,..., from $j = j^{c_{\max}}_{\min}(= j^{c_{\max}-1}_{\max} + 1)$ to $j^{c_{\max}}_{\max}(= j_{\max})$ to chain $c = c_{\max}$).

Variables

x	location of the new facility, $x = (x_1, x_2)$.
α	quality of the new facility.

Input data

p_i	location of demand point i.
w_i	demand (or buying power) at p_i, $w_i > 0$.
f_j	location of existing facility j.
d_{ij}	distance between demand point p_i and facility f_j, $d_{ij} > 0$.
α_j	quality of facility f_j, $\alpha_j > 0$.
γ_i	weight for the quality of the facilities as perceived by demand point p_i, $\gamma_i > 0$.
$d_i^{\min}$	minimum distance from p_i at which the new facility can be located, $d_i^{\min} > 0$.
$\alpha_{\min}$	minimum level of quality for the new facility, $\alpha_{\min} > 0$.
$\alpha_{\max}$	maximum level of quality for the new facility, $\alpha_{\max} \geq \alpha_{\min}$.
S	region of the plane where the new facility can be located.

Miscellaneous

$g_i(\cdot)$	a continuous non-negative non-decreasing function, which modulates the decrease in attractiveness as a function of distance.