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# The Multiple Trip Vehicle Routing Problem with Backhauls: Formulation and a Two-Level Variable Neighbourhood Search

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ABSTRACT

In this paper a new VRP variant the Multiple Trip Vehicle Routing Problem with Backhauls (MT-VRPB) is investigated. The classical MT-VRP model is extended by including the backhauling aspect. An ILP formulation of the MT-VRPB is first presented and CPLEX results for small and medium size instances are reported. For large instances of the MT-VRPB a *Two-Level VNS* algorithm is developed. To gain a continuous balanced intensification and diversification during the search process VNS is embedded with the sequential VND and a multi-layer local search approach. The algorithm is tested on a set of new MT-VRPB data instances which we generated. Interesting computational results are presented. The *Two-Level VNS* produced excellent results when tested on the special variant of the VRPB.



## 1. Introduction

We introduce a new vehicle routing problem (VRP) variant called the Multiple Trip Vehicle Routing Problem with Backhauls (MT-VRPB). The MT-VRPB combines the characteristics of the classical versions of two VRP problems studied in the literature, i.e., the MT-VRP in which a vehicle may perform several routes (trips) within a given time period; and the vehicle routing problem with backhauls (VRPB) in which a vehicle may pick up goods to bring back to the depot once the deliveries are made. Therefore in the MT-VRPB a vehicle may not only perform more than one trip in a given planning period but it can also collect goods in each trip. Since the MT-VRP and the VRPB have been studied independently in the literature, we first provide a brief description of these two routing problems.

### 1.1. MT-VRP

The MT-VRP model is an extension of the classical VRP in which a vehicle may perform several routes (trips) within a given time period. Along with the typical VRP constraints an additional aspect is included in the model which involves the assignment of the optimised set of routes to the available fleet [35]

### 1.2. VRPB

The VRPB is also an extension to the classical VRP that involves two types of customers, deliveries (linehauls) and pickups (backhauls). Typical additional constraints include: (i) each vehicle must perform all the deliveries before making any pickups; (ii) routes with only backhauls are disallowed, but routes with only linehauls can be performed [11].

Both the MT-VRP and the VRPB are considered to be more valuable than the classical VRP in terms of cost savings and placing fewer numbers of vehicles on the roads. These features are very important from both the managerial and the ecological perspectives. By combining the aspects of the above two models into a new model, the MT-VRPB, we achieve a more realistic model. To our knowledge, this is the first time this variant is being studied in the literature. However, there is one study that deals with time windows MT-VRPB-TW by Ong and Suprayogi [26] where an ant colony optimisation algorithm is implemented. Below we present a detailed description of our MT-VRPB model.

### 1.3. MT-VRPB

The MT-VRPB can be described as a VRP problem with the additional possibilities of having vehicles involved in backhauling and multiple trips in a single planning period. The objective is to minimise the total cost by reducing the total distance travelled and the number of vehicles used.

*Problem characteristics:*

- A given set of customers is divided into two subsets, i.e., delivery (linehaul) and pickup (backhaul).
- A homogenous fleet of vehicles.
- A vehicle may perform more than one trip in a single planning period.
- All delivery customers are served before any pickup ones.

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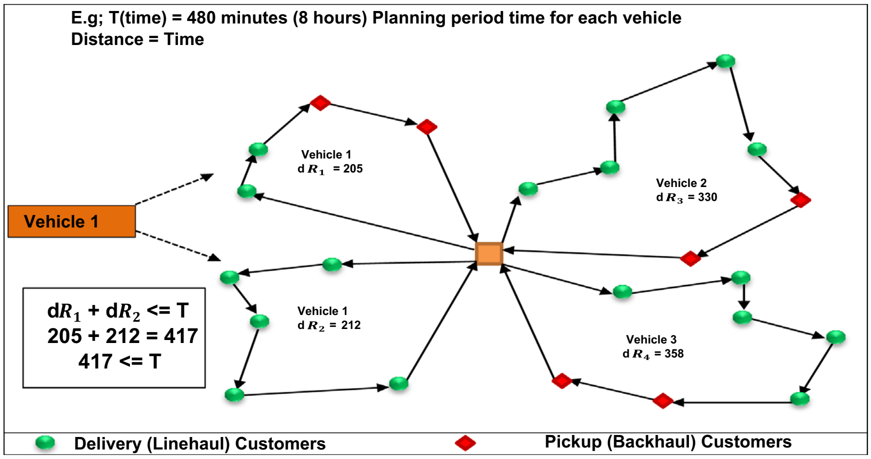


**Fig. 1.** An example of the MT-VRPB.

- Vehicles are not allowed to service only backhauls on any route; however linehaul only routes are allowed.
- Vehicle capacity constraints are imposed.
- Note – the *route length* constraint is not imposed in this study, however the model is flexible to add this constraint if needed.

Fig. 1 presents a graphical example of the proposed MT-VRPB with three homogeneous types of vehicles and a planning period T; *Vehicle 1* performs two trips whereas vehicles 2 and 3 perform one trip each.

The rest of the paper is structured as follows. Section 2 presents the literature review followed by a formulation of the MT-VRPB in Section 3. Section 4 explains the proposed algorithm. The computational results, including the generation of the newly created MT-VRPB data set, are presented in Section 5. Finally, a summary of the conclusions is provided in Section 6.

## 2. Literature review

Since there is no literature available on the MT-VRPB, we provide brief reviews for the two related routing problems namely the MT-VRP and the VRPB.

### 2.1. MT-VRP

The multi-trip vehicle routing was first studied in Salhi [29] where multiple trips were conducted in the context of vehicle fleet mix. Limited to double trips, a matching algorithm is proposed to assign routes to vehicles within a refinement process. Taillard et al. [35] introduced the MT-VRP model based on the classical VRP and proposed a three-phase heuristic algorithm. In the first phase, tabu search is used to generate a population of routes satisfying the capacity constraint; a set of different VRP solutions is then obtained in phase two. Routes are then assigned to the vehicles by solving the bin-packing problem (BPP) in the last phase. Moreover, a set of classical MT-VRP instances are generated in their study which are widely used in the literature as benchmarks. Brandao and Mercer [4] studied a real world application of MT-VRP with time windows and heterogeneous fleet, and used a tabu search algorithm to solve the problem. The methodology developed in this study is adapted in Brandao and Mercer [5] where classical MT-VRP instances were solved and compared. Petch and Salhi [27] developed a multi-phase constructive heuristic algorithm with an objective of minimising the overtime used in multi-trips. The algorithm obtained an MT-VRP solution by solving BPP which is improved further using the 2-Opt and 3-Opt exchange heuristic procedures. Salhi and Petch [31] revisited their previous study described above by using a genetic algorithm which proved to be faster. Olivera and Viera [25] studied this problem and proposed an adaptive memory programming (AMP) approach with tabu search. A set of elite routes is selected randomly from the memory and packed into vehicles solving the BPP while applying some local search refinements based on reducing the driver overtime. The AMP algorithm found feasible packing of bins (without overtime) for most of the classical benchmark instances as compared to the previous studies. Alonso et al. [1] studied a variant of multi-trip called site-dependent periodic MT-VRP using a tabu search algorithm. In this situation, given a planning horizon of *t* days, each customer gets served up to *t* times. Macedo et al. [21] introduced the time windows aspect into this problem and solved the resulting model to optimality. Mingozzi et al. [18] developed an exact method based on two set-partitioning formulations to tackle the MT-VRP. A subset of 52 instances, ranging in size from 50 to 120 customers is tested and 42 are solved to optimality. For the rest, upper bounds are provided. Azi et al. [2] recently proposed an adaptive large neighbourhood search algorithm that makes use of the ruin-and-recreate principle for the MT-VRP with the presence of service time at each node. Cattaruzza et al. [7] proposed a hybrid genetic algorithm for the MT-VRP that uses some adaptations from the literature. A new local search operator called the combined local search (CLS) is introduced that combines the standard VRP moves and performs the reassignments of trips to vehicles by using a swapping procedure leading to good quality results. Cattaruzza et al. [8] then extended the previous model to include time windows using an iterated local search methodology to solve the problem.

It is worth noting that the early studies on the MT-VRP concentrated mostly on the modelling side of the problem and the later ones on the design of powerful methods. By extending the MT-VRP model we aim to break this gap in the literature and open a new research avenue.

Finally, we note that the MT-VRP may form part of more complex logistics problems. Of particular note is the location-routing-scheduling problem, also known as the location-routing problem with multiple trips. This was introduced by Lin et al [15], and solved using simulated annealing. Lin and Kwok [16] extended this model to cater for multiple objectives. Recently, Macedo et al. [22] developed a variable neighbourhood search algorithm for this problem.

### 2.2. VRPB

The VRPB has also attracted a good attention in the literature. Among exact approaches, Yano et al. [37] developed a branch-and-bound framework based on the set covering approach for trucks in a retail chain industry. Toth and Vigo [33] proposed a consolidated