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Solving multifacility Huff location models on networks using metaheuristic and exact approaches

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ABSTRACT

In this paper we consider multifacility Huff facility location problem on networks. First, we introduce a slight modification of the existing mixed integer nonlinear mathematical model and confirm its validity by using the solver for nonlinear optimization, KNITRO. Second, since the problem is NP-hard, we develop three methods that are based on three metaheuristic principles: Variable Neighborhood Search, Simulated Annealing, and Multi-Start Local Search. Based on extensive computational experiments on large size instances (up to 800 customers and 100 potential facilities), it appears that VNS based heuristic outperforms the other two proposed methods.

1. Introduction

Location optimization problems on a network in a competitive environment have been extensively studied in operational research. Hakimi [4] formulated the competitive problem under the assumption that consumers deterministically choose the nearest store. In the real world, this assumption is not always acceptable because consumers do not usually choose the nearest store, they rather choose probabilistically among several stores. This probabilistic choice behavior is modeled by Huff, known as the Huff model [5]. Huff formulated a model for capturing market share, assuming that the probability of a consumer patronizing a shopping center is proportional to its attractiveness and inversely proportional to a power of the distance needed for a consumer to reach it. Although the original Huff model was based on an assumption that a market area is represented by a continuous plane with Euclidean distance, Okabe and Kitamura [10] extended it to the network Huff model by using the shortest path distance on a network. Ghosh et al. [3] considered the problem under the same assumption but for discrete demand (nodal demand). Okunuki and Okabe [11] considered link based demand with slightly changed objective function.

In this paper we apply the network Huff model to a competitive location problem, optimizing new facility locations on a network. We assume that new facilities can be located at any point on the network, and that the demand is generated in the vertices. We introduce a slight modification of the nonlinear mathematical model proposed earlier in [13]. As a step forward with respect to [13], we implemented the model. The implementation was performed by KNITRO software package for solving nonlinear optimization problems, and our computational experience is reported, as well. We considered three different metaheuristics for solving this problem: Variable Neighborhood Search, Simulated Annealing and Multi-Start Local Search metaheuristics for solving this problem. An ampler number of test instances than in [12] is considered and detailed results of the extensive computational testing are shown, as well.

2. Problem formulation

We assume that customers are located in the vertices of a network $\mathcal{N}=(V,E)$, $V=\{v_1,\dots,v_n\}$, $E\subseteq V^2$. The customers make demand. Further, we assume that there are q facilities already located on the network. The facilities provide service and satisfy the demand. They are located at points $y_1,\dots,y_q$ on network $\mathcal{N}$. Hence, the facility locations can be network vertices, as well as other points along the edges. Adopting the notation that $w_i=w(v_i)$

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is the demand associated with vertex v_i, $i \in \{1, \ldots, n\}$, we assume the following properties:

1. $w_i \geq 0$ and
2. $\sum_{i=1}^{n} w_i = 1$.

The demand w may vary from one vertex to another one. For instance, if the demand among the vertices is considered as a random variable, its distribution can be uniform.

Our goal is to *locate p new facilities* $x_1, \ldots, x_p$ on the network, which will respond to the demand made by customers, so that *the captured demand is maximal.*

To state the above location optimization problem more explicitly, let us formulate the network Huff model on $\mathcal{N}$. Firstly, let us introduce *facility attractiveness*, a property assigned to each facility in the system. Facility attractiveness of a specific facility is a scalar, defining the power of the facility to attract customers. It is not related to the location of a facility, yet, it reflects the rating of the facility. It may be measured by the floor area, by the number of services/items that specific facility offers, by the quality of service, by the level of service updating or in any other predefined way. Therefore, let us denote by $a_{y_1}, \ldots, a_{y_q}$ and $a_{x_1}, \ldots, a_{x_p}$ the attractiveness of the existing and new facilities, respectively. In order to unify the notations and to simplify formulas, let us denote by a_{f_j} either

- the attractiveness of the existing facility, when $f \equiv y$ and $j \in \{1, \ldots, q\}$, or
- the attractiveness of the new facility, when $f \equiv x$ and $j \in \{1, \ldots, p\}$,

located at point f_j. Let $d(v_i, f_j)$ be the distance from the customer located in vertex v_i to the facility at f_j on network $\mathcal{N}$. Let us now introduce *the distance deterrence function* $F(d(v_i, f_j))$ which, actually, involves the distance $d(v_i, f_j)$ between the customer in v_i and the facility at f_j. The distance deterrence function is a monotonically decreasing function with respect to $d(v_i, f_j)$. In his original model, Huff specified the distance deterrence function F as a power function, i.e.

$$F(d(v_i, f_j)) = d(v_i, f_j)^{-\lambda}, \quad \lambda > 0. \tag{1}$$

Eventually, let $P(v_i, f_j)$ be the probability of a customer in v_i choosing facility at f_j among the $q+p$ possible facilities. On these terms, the network Huff model is as follows:

$$P(v_i, f_j) = \frac{a_{f_j} d(v_i, f_j)^{-\lambda}}{\sum_{f_k} a_{f_k} d(v_i, f_k)^{-\lambda}}. \tag{2}$$

Using the network Huff model, we proceed with formulating a problem for obtaining the demand $D(f_j)$ captured by facility at f_j. Let $D(v_i, f_j)$ be the demand in v_i captured by facility at f_j. Since the Huff model gives the probability of the customer in v_i choosing the facility at f_j, $D(v_i, f_j)$ is obtained from multiplying the probability $P(v_i, f_j)$ by $w(v_i)$, i.e.

$$D(v_i, f_j) = P(v_i, f_j) w(v_i) = \frac{a_{f_j} d(v_i, f_j)^{-\lambda}}{\sum_{f_k} a_{f_k} d(v_i, f_k)^{-\lambda}} w(v_i). \tag{3}$$

To obtain the demand $D(f_j)$ captured by facility at f_j, we need to sum Eq. (3) over all vertices $v_i \in V$, i.e.

$$D(f_j) = \sum_{v_i \in V} D(v_i, f_j) = \sum_{v_i \in V} \frac{a_{f_j} d(v_i, f_j)^{-\lambda}}{\sum_{f_k} a_{f_k} d(v_i, f_k)^{-\lambda}} w(v_i). \tag{4}$$

With q existing facilities located at points $y_1, \ldots, y_q$ of network $\mathcal{N}$, we are supposed to locate p new facilities at points $x_1, \ldots, x_p$ in order to compete them and capture maximal demand. The total demand captured only by new facilities is given by the formula

$$\sum_{j=1}^{p} D(x_j) = \sum_{j=1}^{p} \sum_{v_i \in V} \frac{a_{x_j} d(v_i, x_j)^{-\lambda}}{\sum_{f_k} a_{f_k} d(v_i, f_k)^{-\lambda}} w(v_i), \tag{5}$$

where $f \in \{y, x\}$; $k \in \{1, \ldots, q\}$ if $f = y$, and $k \in \{1, \ldots, p\}$ if $f = x$. Since it has to be maximal, the problem we have to solve is

$$\max_{x_1, \ldots, x_p \in \mathcal{N}} \sum_{j=1}^{p} \sum_{v_i \in V} \frac{a_{x_j} d(v_i, x_j)^{-\lambda}}{\sum_{f_k} a_{f_k} d(v_i, f_k)^{-\lambda}} w(v_i). \tag{6}$$

3. A mathematical model for the Huff location problem

In this section we discuss the mathematical programming model for the Huff location problem. Let $V = \{v_1, \ldots, v_n\}$ and $E = \{e_1, \ldots, e_m\}$ be a vertex set and an edge set of a network, respectively. If $l : E \longrightarrow \mathbb{R}$ is a weight function defining edge lengths, let $l_i = l(e_i)$ be the length of edge e_i. Since the edge lengths of the graph are known in advance as input data, all pair shortest path distances can be precalculated and considered as input data, too. Therefore, let $d(v_i, v_j)$ be the shortest path distance between vertices v_i and v_j, $\forall i, j \in \{1, \ldots, n\}$.

The location of any point of the graph is given by a triple (v_j, v_k, y), where

- v_j and v_k are endpoints of edge containing the point,
- y is the relative position of the point on edge (v_j, v_k) with respect to edge end v_j.

Let us assign a point to every pair of vertex v and edge $e = (u_e, v_e)$, so that being on the edge e, it is on the largest distance from vertex v. In other words, the distance between the assigned point and vertex v is larger than the distance between vertex v and any other point on the edge e. Relative position M_{ve} of this point on the edge, with regard to preselected endpoint of the edge e, can be expressed as a number from $[0, 1]$. Denote with $dist_{ve}$ the distance between vertex v and the assigned point.

The location of these points are graph properties, therefore, they can be precalculated and considered as input data, as well as their distances $dist_{ve}$ from the corresponding vertex v.

Let us now introduce binary variables x_{fe} (where f is a facility and $e \in E$ is an edge) whose meaning is given with:

$$x_{fe} = \begin{cases} 1, & \text{if facility with index } f \text{ is on edge } e, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Also, we introduce variables y_f whose value is the relative position of facility f on an edge chosen for the facility to be located on. In this context, the shortest path distance $d_{v,f}$ between facility f on edge e and vertex v is:

$$d_{v,f} = dist_{ve} - |M_{ve} - y_f| \, l(e). \tag{8}$$

On the other hand, if facility f is not located on edge e' then, the distance between vertex v and facility f can be described with the inequality:

$$d_{v,f} \geq dist_{ve'} - |M_{ve'} - y_f| \, l(e') - (1 - x_{fe'}) S, \tag{9}$$

where S is a very big number (for example, greater than the sum of lengths of all edges in the graph).

Also, we must bound from above these distances in the following way:

$$d_{v,f} \leq dist_{ve'} - |M_{ve'} - y_f| \, l(e') + (1 - x_{fe'}) S. \tag{10}$$

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