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New heuristic algorithms for discrete competitive location problems with binary and partially binary customer behavior

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ABSTRACT

We consider discrete location problems for an entering firm which competes with other established firms in a market where customers are spatially separated. In these problems, a given number of facility locations must be selected among a finite set of potential locations. The formulation and resolution of this type of problem depend on customers' behavior. The attraction for a facility depends on its characteristics and the distance between the facility and the customer. In this paper we study the location problem for the so-called Binary and Partially Binary Rules, in which the full demand of a customer is served by the most attractive facility, or by all the competing firms but patronizing only one facility of each firm, the one with the maximum attraction in the firm. Two new heuristic algorithms based on ranking of potential locations are proposed to deal with this sort of location problems. The proposed algorithms are compared with a classical genetic algorithm for a set of real geographical coordinates and population data of municipalities in Spain.

1. Introduction

The location of facilities is a strategic decision for a firm that competes with other firms to provide goods or services to the customers in a given geographical area. Different location models and solution procedures have been proposed to cope with these problems which vary depending on the ingredients to be considered, such as location space, facility attraction, customer patronizing behavior, demand function, decision variables, etc. (see for instance survey papers [1–4]).

Most of the models in the literature deal with the location problem for an entering firm that must compete for the market share in a certain region where other firms are already offering the same goods or service. The entering firm is aimed at determination of the optimal locations for the new facilities with respect to maximization of the market share or profit, taking into account the patronizing behavior of customers. Traditionally it is assumed that the customers choose the nearest facility to be served, but, in addition to the distance, the customer can take into account some characteristics of the facilities for its choice.

Some variants of the attraction model, proposed by Huff [5], have been used as customers' choice rules in order to estimate the market share captured by the competing facilities. In this type of models, the attraction of a facility is measured by a parameter, called the facility quality, divided by a non-negative non-descending function of the distance between the customer and the facility. The quality of each facility depends on the characteristics of the facility. The most common customer choice rules are the ones called proportional and binary (see [6]). Following the proportional rule the customers patronize all the facilities in proportion to facility attraction (see for instance [7–9]). In the case of binary rule the customer patronizes the most attractive facility (see [10–14]).

In this paper we will consider a different rule of customers' choice – the Partially Binary Rule – for which the location problem has been little studied in the location literature. In this case, several firms are presented in the market and each firm may own more than one facility. The full demand of a customer is served by all the competing firms, but patronizing only one facility of each firm – the one with maximum attraction – being the demand split among those facilities proportionally to its attraction (see [15]). Some papers refer to this customer choice rule, but it is not studied in depth. In [16] six scenarios resulting from the combination of three customer choice rules (binary, partially binary, and proportional) with two types of services (essential and unessential) are considered on networks, where known discretization results about the existence of a solution for the set of nodes are extended, and recently, Biesinger et al. [17] study the same six scenarios but on discrete space for the leader–follower problem, proposing formulations as MILP problems for both the leader and the follower problems.

Due to the difficulty of these problems, in this paper two heuristic

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algorithms are proposed, which could be used also to solve other discrete competitive location problems. To check performance of the proposed heuristic algorithms, it is necessary to know the optimal solution of the problems in order to compare it with the solution given by a heuristic algorithm. The performance of the proposed heuristic algorithms will be justified by solving the location problem with the binary and partially binary rules, since both problems can be formulated as Integer Linear Programming (ILP) problems, and the optimal solutions can be obtained using standard optimization software (CPLEX, Gurobi, Mosek, Xpress-MP, others), at least for small size problems.

The reminder of the paper is organized as follows: Section 2 consists of description of the location problems, Section 3 is devoted to presentation of the new heuristic algorithms, and Section 4 includes the description and discussion of the experimental investigation of the proposed algorithms; finally, conclusions are presented in Section 5.

2. Discrete location models

An entering firm wants to set up new facilities in a geographical region where similar facilities of other competing firms are already present. There is a set of spatially separated markets and customers are aggregated to geographic demand points in order to make the problem computationally tractable (see [19] for demand aggregation). It is assumed that customers' demand is fixed and known.

The following general notation is used:

Indices:

i, I	index and set of demand points (customers)
k, K	index and set of firms
j, J_k	index and sets of existing facilities of each firm k

Data:

w_i	demand at i
d_{ij}	distance between demand point i and facility j
a_{ij}	attraction that demand point i feels for facility j
$a_i(J_k)$	maximum attraction that i feels for facilities of the existing firm k, $a_i(J_k) = \max\{a_{ij}: j \in J_k\}$
L	a set of candidate locations for the new facilities
s	a number of new facilities to be located

Variables:

X	a set of locations for the new facilities

Next we will review the location problem with the binary choice rule for which an ILP formulation is available and then we will consider the location problem with the partially binary choice rule.

2.1. Model with the binary rule

Following the binary rule of customers' choice, the full demand of a customer is satisfied by only one facility – the one with maximum attraction – but it may occur that there are more than one facility with maximum attraction. If all the tied facilities are owned by the entering firm, then the firm captures the full demand of the customer, but if a part of the tied facilities are owned by its competitors, it is assumed that the entering firm captures a fixed proportion of customer's demand. Finally, if none of the tied facilities are owned by the entering firm, then no demand is captured from the customer.

Let us define the following sets:

$$I^{>} = \{i \in I: a_i(X) > \max\{a_i(J_k): k \in K\}\}$$

and

$$I^{=} = \{i \in I: a_i(X) = \max\{a_i(J_k): k \in K\}\}$$

where $a_i(X) = \max\{a_{ij}: j \in X\}$ is the maximum attraction that point i feels for the new facilities.

The market share captured by the entering firm for the binary rule is:

$$M_b(X) = \sum_{i \in I^{>}} w_i + \sum_{i \in I^{=}} \theta_i w_i \quad (1)$$

where θ_i is the proportion of demand captured from the customer i. The location problem is:

$$(P_b): \max\{M_b(X): |X| = s, X \subset L\} \quad (2)$$

The latter problem can be formulated as an ILP problem considering the following sets and variables:

$$L_i^{>} = \{j \in L: a_{ij} > \max\{a_i(J_k): k \in K\}\} L_i^{=} = \{j \in L: a_{ij} = \max\{a_i(J_k): k \in K\}\}$$

$$I^* = \{i \in I: L_i^{>} \cup L_i^{=} \neq \emptyset\} x_j = \begin{cases} 1 & \text{if a new facility is located at } j \\ 0 & \text{otherwise} \end{cases}$$

$$j \in L y_i = \begin{cases} 1 & \text{if the customer } i \text{ is fully captured by the entering firm} \\ 0 & \text{otherwise} \end{cases}$$

$$i \in I^* z_i = \begin{cases} 1 & \text{if the customer } i \text{ is partially captured by the entering firm} \\ 0 & \text{otherwise} \end{cases} \quad i \in I^*$$

Then, the location problem (P_b) is equivalent to:

$$\begin{aligned} \max \quad & \textstyle\sum_{i \in I^*} w_i y_i + \sum_{i \in I^*} \theta_i w_i z_i \\ \text{s.t.} \quad & y_i + z_i \le 1, \ i \in I^* \\ & y_i \le \textstyle\sum_{j \in L_i} > x_j, \ \forall\, i \in I^* \\ & z_i \le \textstyle\sum_{j \in L_i} = x_j, \ \forall\, i \in I^* \\ & \textstyle\sum_{j \in L} x_j = s \\ & x_j \in \{0, 1\}, \forall\, j \in L, \ y_i, z_i \ge 0, \forall\, i \in I^* \end{aligned}$$

This problem can be solved by standard ILP software, at least for small size problems.

2.2. Partially binary model

In this case, the full demand of the customer is served by all firms, but the customers patronize only one facility from each firm – the one with the maximum attraction. Then the demand is split between those facilities in proportion with their attraction. In this model it is not necessary to consider the possibility of ties because they are irrelevant when it comes to obtaining the total market share captured by the entering firm. It is required to know the number of firms operating on the market, and the number of facilities owned by each firm. This information lets us know the number of facilities to split the demand of each customer, and it also lets us evaluate the proportion of customer demand that will get the most attractive facility of each firm.

The market share captured by the entering firm for the partially binary rule is:

$$M_{pb}(X) = \sum_{i \in I} w_i \frac{a_i(X)}{a_i(X) + \sum_{k \in K} a_i(J_k)} \quad (3)$$

and the location problem is:

$$(P_{pb}): \max\{M_{pb}(X): |X| = s, X \in L\} \quad (4)$$

This problem is a non-linear model, but recently Biesinger et al. [17] have proposed a linear transformation of this model where only two firms are considered, the leader and the follower. That linear transformation follows the idea suggested by Kochetov et al. [20] to obtain a linear formulation of the model when the proportional customer choice rule is considered. In this paper, we have updated the linear transformation to more than two competing firms.

Three new kinds of variables are introduced: