Computers & Operations Research ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

An evolutionary approach to generalized biobjective traveling salesperson problem

Murat Köksalan [a], Diclehan Tezcaner Öztürk [b,*]

[a] Department of Industrial Engineering, Middle East Technical University, Ankara, Turkey
[b] Department of Industrial Engineering, TED University, Ankara, Turkey

ARTICLE INFO

Article history:
Received 30 September 2015
Received in revised form
22 March 2016
Accepted 26 April 2016

Keywords:
Evolutionary computations
Multiobjective decision making
Combinatorial optimization
Multiobjective traveling salesperson problem

ABSTRACT

We consider the generalized biobjective traveling salesperson problem, where there are a number of nodes to be visited and each node pair is connected by a set of edges. The final route requires finding the order in which the nodes are visited (tours) and finding edges to follow between the consecutive nodes of the tour. We exploit the characteristics of the problem to develop an evolutionary algorithm for generating an approximation of nondominated points. For this, we approximate the efficient tours using approximate representations of the efficient edges between node pairs in the objective function space. We test the algorithm on several randomly-generated problem instances and our experiments show that the evolutionary algorithm approximates the nondominated set well.

1. Introduction

The generalized multiobjective traveling salesperson problem (MOTSP) finds efficient tours that visit all the nodes and return to the starting point under multiple objectives. In this problem, we need to determine both the order of visiting the nodes and the edge to use between consecutive node pairs. Finding the efficient edges between node pairs is a multiobjective shortest path problem (MOSPP). The overall problem can be considered as a generalized MOTSP with multiple efficient edges between nodes.

The generalized MOTSP can be considered in many domains; route planning of air vehicles, trucks, trains, and vessels. For the route planning problem of trains, the terminals can be considered as nodes and the alternative paths between terminals can be considered as edges between node pairs. For the route planning problem of air vehicles, the targets they aim to visit can be considered as nodes and the alternative paths between target pairs can be considered as edges. Minimizing total distance traveled, fuel consumption, detection avoidance, duration of travel are some of the objectives that can be used.

In the MOTSP literature, each pair of nodes has been assumed to be connected by a single edge. In a multiobjective context, however, there are typically many efficient edges between pairs of nodes; each edge representing a different tradeoff between objectives. A more general and realistic MOTSP is to consider the efficient tours that are composed of efficient edges. The differences of MOTSP with multiple efficient edges and classical MOTSP are discussed in Tezcaner and Köksalan [20].

The classical MOTSP and MOSPP are NP-hard [4]. There are a number of studies that develop heuristics for the classical MOTSP. Paquete and Stützle [17] develop a two-phase local search method, Jaszkiewicz and Zielniewicz [6] develop an algorithm that uses path relinking and Pareto local search, Lust and Teghem [13] propose a new two-phase Pareto local search for MOTSP and Lust and Jaszkiewicz [12] develop speed-up techniques for this heuristic for large MOTSPs. Ke et al. [7] propose a memetic algorithm for multiobjective combinatorial optimization problems and apply the algorithm to the MOTSP. Özpeynirci and Köksalan [15,16] study MOTSPs that have special structures. For a review on heuristics developed for MOTSP, we refer the reader to Lust and Teghem [13]. The generalized biobjective traveling salesperson problem (BOTSP) is studied in Tezcaner and Köksalan [20] and Tezcaner Öztürk and Köksalan [21]. They develop interactive algorithms that find the most preferred solution of a decision maker (DM) under linear preference functions [20] and quasiconvex preference functions [21]. They apply the algorithms on generalized BOTSPs.

In this study, we address the generalized BOTSP with a heuristic approach. We need to solve both the biobjective shortest path problem (BOSPP) and the BOTSP with multiple efficient edges between nodes. Since both BOTSP and BOSPP have been shown to be NP-hard, heuristic algorithms are required for addressing large instances. We develop an approach that generates an approximation for the nondominated set of the generalized BOTSP.

* Corresponding author.

http://dx.doi.org/10.1016/j.cor.2016.04.027
0305-0548/

Please cite this article as: Köksalan M, Tezcaner Öztürk D. An evolutionary approach to generalized biobjective traveling salesperson problem. Computers and Operations Research (2016), http://dx.doi.org/10.1016/j.cor.2016.04.027

We define the problem in Section 2 and develop the algorithm in Section 3. We give computational results in Section 4 and present our conclusions in Section 5.

2. Problem definition

We first present some notation and definitions that are adapted from Tezcaner and Köksalan [20].

Let x denote the decision variable vector, X denote the feasible set, Z denote the image of the feasible set in objective function space, and point $z(x)=(z_1(x), z_2(x),\ldots,z_p(x))$ be the objective function vector corresponding to the decision vector x, where p is the number of objectives and $z_k(x)$ is the performance of solution x in objective k. We assume, without loss of generality, that all objectives are to be minimized. A solution $x\in X$ is said to be *efficient* if there does not exist $x'\in X$ such that $z_k(x')\le z_k(x)$ $k = 1, \ldots, p$ and $z_k(x') < z_k(x)$ for at least one k. If there exists such an x', x is said to be *inefficient*. The set of all efficient solutions constitute the efficient set.

If x is efficient, then $z(x)$ is said to be *nondominated*, and if x is inefficient, $z(x)$ is said to be *dominated*. The set of nondominated points constitute the nondominated set. A nondominated point $z(x)$ is a *supported nondominated point* if there exists a positive linear combination of objectives that is minimized by x. Otherwise, $z(x)$ is an *unsupported nondominated point*. We define an *extreme nondominated point* as a supported nondominated point that has the minimum possible value in at least one of the objectives.

For the generalized BOTSP, we first find all efficient edges between node pairs utilizing algorithms developed for BOSPP. MOSPP has been studied well in the literature (see for example [5,18]). Evolutionary algorithms (EAs) have also been developed for MOSPP (see for example [14]). After finding all efficient edges connecting each node pair, we solve the formulation given below for the generalized BOTSP (as in [21]) to find efficient tours that use a subset of the efficient edges.

Let $G=(N,E)$ be an undirected graph with node set $N=\{1,2,\ldots\}$ and edge set E, and let R_{ij} be the index set of efficient edges between node pair (i,j). Let the binary decision variable x_{ijr} take value 1 if the rth efficient edge connecting nodes (i,j) is used, and 0 otherwise for $r\in R_{ij}$, c_{ijr}^k denote the kth objective value of the rth efficient edge between nodes i and j, and $P=\{(i,j)|i\in N, j\in N, i\ne j\}$ be the set of all node pairs. The formulation of the problem is as follows:

$$\text{Min } z_1(x)=\sum_{(i,j)\in P}\sum_{r\in R_{ij}} c_{ijr}^1 x_{ijr} \quad (1)$$

$$\text{Min } z_2(x)=\sum_{(i,j)\in P}\sum_{r\in R_{ij}} c_{ijr}^2 x_{ijr} \quad (2)$$

Subject to:

$$\sum_{j\in N}\sum_{r\in R_{ij}} x_{ijr}=1 \;\; i\in N \quad (3)$$

$$\sum_{i\in N}\sum_{r\in R_{ij}} x_{ijr}=1 \;\; j\in N \quad (4)$$

$$\sum_{i\in U}\sum_{j\in N/U}\sum_{r\in R_{ij}} x_{ijr}\ge 1 \;\; U\subset N, 2\le |U|\le |N|-2 \quad (5)$$

$$x_{ijr}\in\{0,1\} \;\; (i,j)\in P,\; r\in R_{ij} \quad (6)$$

Here we assume that $c_{ijr}^k\ge 0$ for $k=1,2$ and $r\in R_{ij}$; i.e. the values of the edges constituting these two objectives are assumed to take nonnegative values. Eq. (3) ensures that one edge leaving each node is selected and Eq. (4) guarantees an edge entering each node is selected. Constraint (5) is a standard traveling salesperson problem subtour elimination constraint that assures at least one connection between two separate subsets of the nodes.

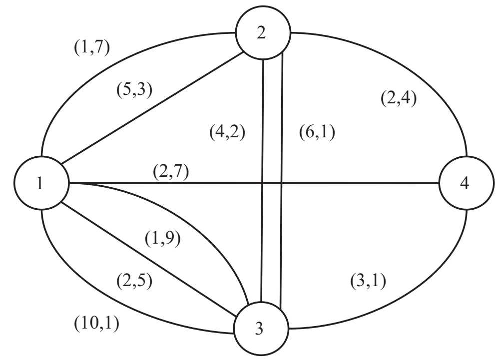

Fig. 1. Generalized BOTSP example.

An efficient solution corresponds to the efficient tour of the traveling salesperson problem (TSP) together with the specific efficient edges used between the consecutive nodes visited by that tour. Moving from node $i\in N$ to node $j\in N$, there may be many efficient edges. Let π be a cyclic permutation of set N and Π be the set of all tours. As we have many edge options between the nodes, all $\pi\in\Pi$ can be traversed by a number of combinations of the efficient edges between its connected nodes. In the rest of the paper, to differentiate among different edge options between two nodes, we denote the rth efficient edge between nodes i and j as e_{ijr} where $r\in R_{ij}$ and the set of edges as $E=\left\{e_{121}, e_{122},\ldots,e_{|N|,|N|-1,|R_{|N|,|N|-1}|}\right\}$. Similarly, we denote the uth efficient edge combination for tour π as $t_{\pi u}$ where $u\in U_\pi$, and its kth objective function value as $C_{\pi u}^k$, for $k=1,2$. To demonstrate, consider the example in Fig. 1 with four nodes. The values written in parentheses next to each edge are the first and second objective function values, respectively, of the corresponding efficient edge. Between node pairs 1-4, 2-4, and 3-4, we have single connections (single efficient edges). Therefore, $|R_{14}|=|R_{24}|=|R_{34}|=1$. There are two efficient edges between node pairs 1-2 and 2-3 ($|R_{12}|=|R_{23}|=2$) and three efficient edges between node pair 1-3 ($|R_{13}|=3$). For example, the objective function values for the efficient edge between node pair 1-4 are $c_{14,1}^1=2$ and $c_{14,1}^2=7$, using our notation. We have three possible distinct tours to visit the four nodes, $\Pi=\{$1-2-3-4-1, 1-2-4-3-1, 1-3-2-4-1$\}$. For tour 1-2-3-4-1, there are four different routes considering the different edges between the nodes (all possible combinations of the two edges between node pair 1-2, two edges between node pair 2-3, one edge between node pair 3-4, and one edge between node pair 4-1). When all these combinations are enumerated and their objective function values are calculated, all four points turn out to be nondominated with the following objective function values: $C_{1-2-3-4-1,1}^1=10$, $C_{1-2-3-4-1,1}^2=17$, $C_{1-2-3-4-1,2}^1=12$, $C_{1-2-3-4-1,2}^2=16$, $C_{1-2-3-4-1,3}^1=14$, $C_{1-2-3-4-1,3}^2=13$, and $C_{1-2-3-4-1,4}^1=16$, $C_{1-2-3-4-1,4}^2=12$. Therefore, the nondominated set of tour 1-2-3-4-1 is composed of four points; (10,17), (12,16), (14,13) and (16,12).

When we enumerate the solutions of tour 1-2-4-3-1 and find their objective function values, we obtain six points, four of which are nondominated, disregarding the nondominated points of other tours. Similarly, for tour 1-3-2-4-1, we obtain six points, five of which are nondominated, again disregarding the nondominated points of other tours. When we evaluate all points of all tours together, some points turn out to be dominated. In our case, the

Please cite this article as: Köksalan M, Tezcaner Öztürk D. An evolutionary approach to generalized biobjective traveling salesperson problem. Computers and Operations Research (2016), http://dx.doi.org/10.1016/j.cor.2016.04.027