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A Box Decomposition Algorithm to Compute the Hypervolume Indicator

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Abstract

We propose a new approach to the computation of the hypervolume indicator, based on partitioning the dominated region into a set of axis-parallel hyperrectangles or boxes. We present a nonincremental algorithm and an incremental algorithm, which allows insertions of points, whose time complexities are $O(n^{\lfloor \frac{p-1}{2} \rfloor+1})$ and $O(n^{\lfloor \frac{p}{2} \rfloor+1})$, respectively, where n is the number of points and p is the dimension of the objective space. While the theoretical complexity of such a method is lower bounded by the complexity of the partition, which is, in the worst-case, larger than the best upper bound on the complexity of the hypervolume computation, we show that it is practically efficient. In particular, the nonincremental algorithm competes with the currently most practically efficient algorithms. Finally, we prove an enhanced upper bound of $O(n^{p-1})$ and a lower bound of $\Omega(n^{\lfloor \frac{p}{2} \rfloor} \log n)$ for $p \geq 4$ on the worst-case complexity of the WFG algorithm.

Keywords: Multi-objective optimization, Hypervolume indicator, Klee's measure problem

1. Introduction

In multi-objective optimization (MOO), since objective functions are often conflicting in practice, there is typically no single solution that simultaneously optimizes all objectives. Instead there are several efficient solutions, i.e. that cannot be improved on one objective without degrading at least another objective. Due to the possible large number of efficient solutions or even nondominated points – their images in the objective space – approximation algorithms are often favored in practice. These algorithms are able to generate several discrete approximations or representations of the nondominated set and quality

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